2. Torque and moment of inertia

Introduction

In this laboratory we will use a falling mass \( m \) to apply a constant net torque to a “dumbbell-shaped” system of masses \( M \) attached to a rotatable post (Fig. 1). We will use a photogate and Data Studio to measure the change in the velocity of a flag at the end of the dumbbell as the mass descends, and from this measurement we can deduce the angular acceleration produced by the torque. Using our knowledge of the physics of rotating systems, we can then determine the moment of inertia of the dumbbell.

The measured moment of inertia can be compared to a calculation of the dumbbell’s moment of inertia.

![Figure 1](image)

The photogate will be used to measure the velocity of the flag, which is the tangential velocity \( v_T \) at the edge of the dumbbell system. To convert the tangential velocity to angular velocity \( \omega \), we use the relation:

\[
(1) \quad v_T = \omega R_F
\]

where \( R_F \) is the distance from the center of the dumbbell system to the position of the flag (Fig. 2).
As the mass descends, the torque $\tau$ applied at the base of the support will cause the tangential velocity to increase. The slope of the best fit line to the velocity vs. time graph will give us the tangential acceleration $a_T$ of the dumbbell system. The relationship between tangential acceleration and angular acceleration $\alpha$ can be found by differentiating Eq. (1) with respect to time:

$$a_T = \alpha R_F$$ \hspace{1cm} (2)

The applied torque $\tau$ is related to the angular acceleration $\alpha$ by the moment of inertia $I$:

$$\tau = I \alpha$$ \hspace{1cm} (3)

The torque is given by:

$$\vec{\tau} = \vec{r} \times \vec{F}$$ \hspace{1cm} (4)

where $r$ is the distance from the center of the dumbbell system to the outside of the central post where the string wrapped and the force is exerted. Since the force and radius vectors are orthogonal,

$$\tau = r F$$ \hspace{1cm} (5)

The force is given by the tension $T$ in the string going from the base of the post through the pulley to the mass $m$. According to Newton’s 2nd Law, the tension in the string is given by

$$T = m(g - a) = F$$ \hspace{1cm} (6)

where $a$ is the acceleration of the descending mass $m$ and $g$ is the acceleration due to gravity. The acceleration $a$ is also equal to the tangential acceleration at the edge of the post:

$$a = \alpha r = a_T \frac{r}{R_F}$$ \hspace{1cm} (7)
Finally we can use the relationship between torque and angular acceleration, Eq. (3), and Eq. (7), to solve for the moment of inertia in terms of quantities we can measure:

$$I = m r^2 \left( \frac{g R_F}{a_T r} - 1 \right)$$  \hspace{1cm} (8)

**Prelab Questions**

1. Derive Eq. (8).

2. Calculate the moment of inertia for two point masses $M$ equidistant from the center of rotation. (This is the simplest model of the dumbbell system). Will the true moment of inertia of the dumbbell system be larger or smaller than this estimate?

**Determining Friction in System**

The first task is to take into account friction in the system. To compensate for the friction of the rod and dumbbell assembly, we will find out how much mass ($M_f$) must be added to the end of the string to overcome kinetic friction and allow the system to rotate at a constant speed. This “frictional” mass $M_f$ will be subtracted from the mass attached to the end of the string $m$ to get a more accurate result. Thus Eq. (8) for determining the moment of inertia will become

$$I = (m - M_f) r^2 \left( \frac{g R_F}{a_T r} - 1 \right)$$  \hspace{1cm} (9)

$M_f$ is between 10-50 grams, so you will need to attach some small masses to the end of the string.

**Computer and Equipment Setup**

1. Connect the Data Studio interface to the computer, turn on the interface, and then turn on the computer.

2. Double click on the Data Studio icon. When the window opens click on **Create Experiment**.

3. Connect the photogate sensor’s plug to Digital Channels 1 of the Data Studio interface.

4. In the Sensors panel on the left, scroll down to **Photogate** and double click. An icon for the photogate sensor will appear in the right panel.

5. Near the top of your screen, click on timer. Under **label** type “time blocked”. Under **timing sequence choices** select “blocked”. Go again to timing sequence choices and now
select “unblocked”. Click “Done”. You have just programmed the computer to measure the length of time that the photogate is blocked by the flag on your dumbbell.

6. In order to calculate the speed of the dumbbell click on Calculate near the top of your screen. For your equation type velocity = 0.025 / x. Select variables → define → data measurement. Now select “time blocked”. Close the window.

7. Double click on Digits in the bottom left panel and select velocity 0.025 / x. Then double click on Graph in the bottom left panel and select x = velocity = 0.025 / x.

**Determining the Frictional Mass**

1. Position the two masses $M$ attached to the dumbbell at positions equidistant from the central rod.

2. Attach some mass $m$ between 10-50 g to the end of the string. Then wind the string up so that the mass is just below the end of the pulley. Hold the dumbbell so that it doesn’t rotate.

3. Click the Monitor button and then give the dumbbell a little push to get it going (remember we’re trying to measure the kinetic friction, not the static friction!). Is the velocity changing or staying constant.

4. Adjust the mass until the velocity is relatively constant. This is your value for the “frictional mass” $M_f$. Record this value in your lab notebook.

**Graph 1.** Print a graph of the velocity of the flag when the frictional mass is attached to the end of the string. It should be relatively flat if you have obtained the right value.

**Finding the Moment of Inertia**

Now we will move on to measuring the tangential acceleration and determining the moment of inertia of the dumbbell system.

1. Attach an $m = 500$g mass to the end of the string. Wind the dumbbell apparatus up so that the mass is near the pulley. Hold the dumbbell in place so it doesn’t start to rotate.

2. Click the Start button to begin recording data and release the dumbbell. Click the STOP button just before the mass $m$ hits the ground.

**Graph 2.** Print the graph of the velocity of the flag as a function of time with the linear fit. Record your value of $a_T$.

Repeat these steps 3 times to find an average $a_T$ and the uncertainty in your measurement.
**Question 1.** Measure the other relevant parameters in Eq. (9) and record them in your notebook. Calculate the moment of inertia $I$ based on these parameters and your average value of $a_T$. How close is your result to your calculation of the moment of inertia for two point masses a distance $R_M$ from the center of the rod you did in Pre-Lab Question 2? Is this difference explained by the statistical uncertainty of your measurement, or is there a systematic effect? Discuss possible systematic effects.

**Different moment of Inertias**

**Prediction 1.** How will the moment of inertia change if the masses $M$ are moved further from the center? If they are moved closer? How does this affect the angular acceleration $\alpha$ for a given mass $m$?

Repeat the above procedure to determine the moments of inertia for at least four different distances of the masses $M$ from the center of the rod. Make a table in your lab notebook of these different values.

**Graph 3.** Plot the moment of inertia of the dumbbell as a function of the distance of the masses $M$ from the center of the rod. Make sure the horizontal axis goes to zero and extrapolate your data to find out what the moment of inertia for the system would be if the mass was concentrated at the center ($R_M = 0$). Call this value of the y-intercept $I_0$.

**Question 2.** How does $I_0$ compare with the difference between your measured $I$ and the moment of inertia for two point masses a distance $R_M$ from the center of the rod you did in Pre-Lab Question 2? Discuss.

**Question 3.** How does the measured dependence of the moment of inertia on the position of the masses $M$ compare with your prediction?