1. Conservation of Mechanical Energy

Introduction

If the only forces acting are conservative forces, i.e., no frictional forces, then mechanical energy of a system is conserved:

\[ E = K + U \]

where

\[ K = \frac{1}{2}mv^2 \quad \text{Kinetic Energy} \]
\[ U_G = mgh \quad \text{Gravitational potential energy} \]
\[ U_S = \frac{1}{2}ky^2 \quad \text{Elastic potential energy of a spring} \]

During the past 150 years the law of energy conservation has been extended to include other forms of energy such as thermal, chemical, electrical, nuclear, radiant, etc. In its more general form the law of conservation of energy states:

Energy cannot be created or destroyed; it may be transformed from one form to another (mechanical, thermal, electrical, nuclear, radiant, etc.), but the total energy in any isolated system never changes.

In this experiment you will study the mechanical energy of a mass attached to a spring. You will verify that the energy of the mass-spring system is indeed conserved. The forms of energy involved are elastic potential energy of the spring \( U_S \), the kinetic energy \( K \) of the mass \( m \), and the gravitational potential energy of the mass, \( U_G \). The total mechanical energy of the system on the right is:

\[ E = K + U_S + U_G = \frac{1}{2}mv^2 + \frac{1}{2}ky^2 + mgh \]

where \( y \) is measured from the equilibrium position of the mass \( m \), the position where \( m \) hangs at rest, and \( k \) is the spring constant.

Objectives

- To review how the displacement of a spring is related to the spring force.
- To practice making a graph and determining both the slope and area under the curve.
- To discover how for a force versus displacement graph of a spring the slope and area under the curve are related to different physical properties of the system.
- To verify the conservation of mechanical energy.
Hooke's Law for a Spring

If a spring is stretched a distance $y$ which is not too large, then Hooke's law states the spring exerts a force $F$ which is proportional to $y$:

$$ F = -k\Delta y \quad (1) $$

where $k$ is the force constant of the spring and the negative sign implies $F$ is in the opposite direction of the displacement $y$. In this section you will calculate the spring constant of your spring by measuring the displacement caused by a known force. Instead of only doing this once, you will collect a series of data points $(F,y)$ and plot them. The slope of the line is the spring constant.

1. Hang the spring vertically, larger end down. Note the position of the bottom of the spring. Now place a 100g mass on the spring and record the displacement of the bottom of the spring.

2. Continue to add mass in increments of 100g until you reach 700g. At each step record the total displacement from the equilibrium position. Make a table with columns for mass (kg) and displacement (meters).

3. At each step the force with which the spring pulls up on the mass will be equal and opposite to the force of gravity pulling down. In a third column calculate the magnitude of the spring force, $|F| = mg$.

4. In your lab book, sketch a hand drawn plot of the spring force $F$ (Newtons) versus the spring displacement $y$ (meters). From equation (1) we can see that the slope of this line is the spring constant $k$. DO NOT USE EXCEL TO MAKE YOUR GRAPH THIS WEEK.

**Question 1.** Does your plot agree with Hooke’s Law (Eq. 1)? What is the value of the spring constant in N/m?
Elastic Potential Energy in the Spring

How much energy is stored in a spring as we stretch it? A load $m$ added to a spring applies a force $mg$ which stretches the spring, storing elastic potential energy. If the added load takes the spring from some initial force $F_1$ at $y_1$ to a final force $F_2$ at $y_2$, then the work done by $mg$ is the area under the graph between $F_1$ and $F_2$, i.e, the shaded area under the line.

1. On your graph draw horizontal dashed lines at the forces corresponding to 0.500 kg and 0.700 kg. Draw vertical dashed lines where these hit your best fit line.

2. Shade in the area between the two vertical dashed lines and below your best fit line. Determine the area of this region. It will be convenient to think of the shaded region as a triangle stacked on top of a rectangle. Calculate the area of each of these regions and add them together. This area is the work done by the mass to stretch the spring, which has been stored as spring potential energy.

**Question 2.** What is the change in spring potential energy $\Delta U_S$ of the spring when the load is increased from 0.500 kg to 0.700 kg? Compare your value of $\Delta U_S$ to the work done by the mass to stretch the spring as determined above. Give your answer in joules.
The Spring in Motion: Energy Conversion

In this section we will stretch the spring with 0.700 kg, then release 0.200 kg of that mass. The extra energy stored in the spring (shaded area in your plot) will be converted into kinetic and gravitational potential energy causing the spring and remaining mass to oscillate. We will measure these other two forms of energy to verify that energy is conserved.

1. Hang a load of 0.500 kg on the spring. Locate the equilibrium position for this load and place a photocell and lamp at this location. Place a small flag on the 0.500 kg mass. Align the flag so that it passes directly between the photocell and lamp when the mass is in equilibrium (figure b below). Record the position of the bottom of the 0.500 kg mass ($h_i$).

2. Hang an extra load of 0.200 kg on the 0.500 kg mass by a thread. Record the position of the bottom of the 0.500 kg mass ($h_i$) when the system is in equilibrium. Calculate the gravitational potential energy that the 0.500 kg mass will gain as it rises from $h_i$ to $h_f$, in Joules.

   \[ \text{Gravitational PE} = mg(h_f - h_i) \quad (m = 0.500 \text{ kg}) \]  

3. Cut the thread and catch the 0.200 kg mass in your hand. The timer will record how long ($t$) the flag took to pass by the photocell. The velocity of the system can be calculated as \( v = \frac{d}{t} \), where \( d \) is the width of the flag. Determine \( v \) in m/s.

4. The kinetic energy of the system will include the kinetic energy of both the 0.500 kg mass and the spring. Since only the bottom of the spring is moving, we only take into account $\frac{1}{3}$...
of the spring mass \((m_s)\). Calculate the total kinetic energy as the system passes through equilibrium, in Joules.

\[
\text{Kinetic Energy} = \frac{1}{2} (0.500 \text{ kg} + \frac{1}{3} m_s)v^2
\]  

(3)

If energy is conserved then we would expect:

\[
\text{Spring PE} = \text{Gravitational PE} + \text{Kinetic Energy}
\]  

(4)

5. You determined the spring potential energy earlier from your graph. What is the sum of the gravitational and kinetic energy? Is it the same as the spring potential energy?

**Question 3.** Is mechanical energy conserved in your experiment? What is the percent discrepancy between the left and right hand side of the total energy equation (4) above? Why might the two sides of the equation be different? Which one would you expect to be smaller?