Indoor Lab: Impact Craters

I. Introduction

- History
  - Kinetic Energy: A famous experiment by Willem Gravesande (summarized by Emile du Chatelet 1749) measured the size of dents made in clay by dropping balls of various masses and heights. They found that the quantity that determined the dent size was (what we now call) the “Kinetic Energy”, which depends upon the mass and square of the velocity: \( K = \frac{1}{2} mv^2 \).
  - Volcanic Model of Craters: Before we went to the moon, the majority of scientists thought that the craters were not formed by impacts, but rather had volcanic origins (either fossil “bubbles” from escaping gas when surface was molten, or similar to caldera craters at the top of volcanoes). Grove Gilbert (in the 1890s) argued against the volcanic model by showing that the floors of the moon’s craters were not elevated, but rather were lower than the surrounding surface of the moon.
  - Impact Model of Craters: The argument against craters being made from impacting asteroids was that they would come in from random directions, and hence most craters would be ellipses rather than circles. Also, no impact craters were seen on the earth (or Mars). It was only 60 years ago that quite simple experiments were done that showed high velocity impacts coming in sideways still make circular craters. Eugene Shoemaker, in his Ph.D. thesis (1960s) argued that the Barringer crater in Arizona was in fact due to a meteor impact. Geology done by astronauts landing on the moon (1969-1972) supports the impact theory.

- Theory
  - Galileo’s law states the a ball dropped from height “h” will obtain velocity \( v = \sqrt{2gh} \), independent of the mass of the ball (g=9.8 m/s²).
  - The Kinetic energy of the ball dropped from height “h” will be: \( K = \frac{1}{2} mv^2 = mgh \).
  - The diameter of a crater “D” is related to the volume “V” of the “hole”: \( \frac{D^3}{V} \propto \). Hence the diameter is proportional to the cube root of the volume: \( D \propto V^{\frac{1}{3}} \).

- Model
  The question is how the volume (actually the diameter) of a crater scales with the energy of the impact. The basic model is that \( D = kE^p \) where “k” and “p” are numeric constants. The two basic models are:
    - Deformation: The crater is made by either vaporizing the material (something we are NOT going to do in lab) or compression. For this process we’d expect the volume to be proportional to the energy, hence the power “p” in the model would be \( p=1/3 \).
    - Excavation: The crater is made by lifting the material out of the hole. Hence the energy needed is due to mechanical work (force times change in height). The force needed is proportional total mass (total volume), and the height is proportional to the diameter of the crater so we expect the energy to be proportional: \( E \propto D^4 \). Hence we expect the diameter to scale to the 4th root of energy, or parameter \( p= \frac{1}{4} \).
II. Setup

- Measure the masses of your balls (report in table!)
  - You have several metal balls, and perhaps some plastic or glass ones.
  - Measure all of their masses

- Sandbox
  - Make sure the sand surface is smooth.
  - A light coming in sideways makes it easier to measure crater diameters.

- Dropping Balls
  - Might have a magnetic device to hold balls.
  - Measure ball heights from surface of sand, NOT from floor!
  - Repeat all measurements 3 times.

- Data Table should include
  - Ball type, its mass (in kg)
  - Height (in meters)
  - Kinetic energy of ball: \( E = \frac{1}{2}mv^2 = mgh \) (units of Joules)
  - Three measurements of crater diameter (in meters) at same height.
  - Average crater diameter (in meters)

III. First Experiment: Energy

- Start with smaller ball (around 30 gm?)
  - Drop from height of 2 meters, measure crater diameter
  - Repeat measurement 3 times
  - Compute average crater diameter
  - Calculate the “Energy” of the ball in this experiment (\( E = mgh \))

- Switch to medium ball (around 45 gm)
  - Calculate the height which will give this ball the same energy. [Hint: if the bigger ball is 3× more massive, you would drop it from 1/3 the height.]
  - Drop this ball from this height (3 times of course!), and measure crater diameter

- Switch to big ball (around 66 gm)
  - Calculate the height which will give this ball the same energy.
  - Drop this ball from this height (3 times of course!), and measure crater diameter

Question 1: Summarize. All three of the ball drops have the same energy. Do they make approximately the same size crater as expected?
**IV. Second Experiment: Power Relationship.**

- **Start with the most massive ball (around 66 gm?)**
  - Drop from height of 2 meters, measure crater diameter
  - Repeat measurement 3 times
  - Compute average crater diameter
  - Calculate the “Energy” of the ball in this experiment \( E = mgh \)

- **Test Excavation Theory**
  - Switch to small ball (around 28 gm?)
  - Calculate the height which will give this ball (1/16) the energy as the heavy ball.
    [Hint: if the bigger ball is 3\( \times \) more massive, you would drop the small ball from 3/16 the height or 0.375 meters.]
  - Drop this ball from this height (3 times of course!), and measure crater diameter
  - If the excavation theory is right, then the diameter of this crater will be half that made by the big ball.

- **Test Deformation Theory**
  - Continue using small ball (around 28 gm?)
  - Calculate the height which will give this ball (1/8) the energy as the heavy ball.
    [Hint: if the bigger ball is 3\( \times \) more massive, you would drop the small ball from 3/8 the height or 0.75 meters.]
  - Drop this ball from this height (3 times of course!), and measure crater diameter
  - If the deformation theory is right, then the diameter of this crater will be half that made by the big ball.

**Question 2:** Summarize. From the above experiments, which model is correct, excavation or deformation? What then is “p” in the model \( D = kE^p \)? (Hint, in which case was the diameter of the crater for the small ball half that of made by the big ball?).

**Question 3:** From your data, what would be the approximate value of “k” in the model \( D = kE^p \)? Be sure to include the proper units.
V. Analysis

- **More Data**
  - If time permits, drop one or more balls from a few different heights to add more data to your table. Make sure you include a full range of heights, from small (0.1 meters) to big (2 meters). Suggest you use numbers something like: \{2, 1.5, 1.0, 0.5, 0.2, 0.1 \} meters.
  - For ALL your data, calculate the energy of the balls

- **Plot 1**
  - Plot average diameter “D” vs Energy “E” (as an “xy scatter plot”)
  - Add “power” trendline (click on options to show equation and Rsquared)
  - The equation will show the values of “k” and “p” in our model.

- **Plot 2**
  - Add a column to your table, calculating the log of the diameter: \( y = \log(D) \)
  - Add a column to your table, calculating the log of the energy: \( x = \log(E) \)
  - Plot \( \log(D) \) vs \( \log(E) \) (as an “xy scatter plot”)
  - Add “linear” trendline (click on options to show equation and Rsquared)
  - In this case the equation will show slope “m” and intercept “b”: \( y = mx + b \).
  - The slope gives you the power parameter “p” (i.e. \( p = m \))
  - From the intercept “b”, calculate: \( k = 10^b \)

**Question 4:** Summarize Plot 1. Does the data fit the model well (i.e. is the value of Rsquared close to 1)? Summarize Plot 2. We expect this to be a line. Is it? (is Rsquared close to 1)? Which graph has the “best fit”?

**Question 5:** From your graphs, what is your value of “p”? How does it compare with question 1? Which model (excavation or deformation) is probably the right one?

**Question 6:** From your graphs, what is your value of “k”? How does it compare with question 3? Hence what is your equation \( D = kE^p \)?

VI. Analysis of Recent Meteor Impact

- **Recent (2013 Feb 13) Chelyabinsk (Russian) Meteor Impact released about \( 2.1 \times 10^{15} \) Joules of energy. Classified as an ordinary type L Chondrite (10% iron).

- Assume that the asteroid was moving at approximately 20 km/sec=20,000 m/s (about twice escape velocity), calculate the mass “m” (in kg) of the asteroid from the equation: \( K = \frac{1}{2} mv^2 \). Compare to estimate of 10,000 Tons (you’ll have to convert units!).

- Assume density \( \rho = 3350 \) kg/m\(^3\), then estimate the diameter (in meters). Note:
  - Knowing mass and density of the asteroid, calculate the volume “V” from: \( m = \rho V \)
  - Note that the volume of a sphere is given: \( V = \frac{4}{3} \pi r^3 \), estimate the radius “r”.
  - What then is your estimate of the diameter of the asteroid?

- Using your equation (question 6), calculate the size of hole it should have made, if it hit in one big piece.

**Question 7:** The hole in the ice of Lake Chebarkul was 6 meters in diameter. Is this consistent with the meteorite breaking up into small pieces, or hitting in one big piece?

==================================================================
Sample Student Data (not very good)

<table>
<thead>
<tr>
<th>Mass (kg)</th>
<th>height (m)</th>
<th>v (m/s)</th>
<th>D1</th>
<th>D2</th>
<th>D3</th>
<th>Avg D</th>
<th>E (J)</th>
<th>Log E</th>
<th>Log D</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.22758</td>
<td>1.64</td>
<td>5.67</td>
<td>0.15</td>
<td>0.145</td>
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<td>0.135</td>
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<td>4.95</td>
<td>0.13</td>
<td>0.1325</td>
<td>0.115</td>
<td>0.126</td>
<td>2.788</td>
<td>0.445</td>
<td>-0.900</td>
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<tr>
<td>0.22758</td>
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<td>4.27</td>
<td>0.12</td>
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<td>0.125</td>
<td>0.123</td>
<td>2.074</td>
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<td>0.93</td>
<td>4.27</td>
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<td>0.455</td>
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<td>0.091</td>
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<tr>
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<td>0.075</td>
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<td>0.099</td>
<td>0.091</td>
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<td>-1.039</td>
</tr>
</tbody>
</table>

The “error” these students made is that they did not have a large range in heights. Note that they only went from 0.93 to 1.76 meters. You should go up to 2 meters, and most certainly include lower heights, such as 0.10, 0.25, 0.5 meters.

The $R^2$ shows the “quality” of the fit. A “perfect” fit would have $R^2=1$, so values of 0.95 are fair. A terrible fit has a low $R^2$ of near zero. Note that we expect that you can get $R^2 > 0.97$ for this experiment, otherwise you’ve probably done something wrong.

Note that the $R^2$ on both graphs should be the same, as should the parameter “p” [here $p=0.2398$]. The first graph shows $k=0.1042$ while the second graph shows $b=-0.9821$ which looks different, but calculating: $k = 10^p = 10^{-0.9821}$ will give the same value.

Hence the model is: $D = kE^p$, where $p=0.2398$ and $k=0.1042$. 

\[
y = 0.1042x^{0.2398}
\]

\[
R^2 = 0.9534
\]

\[
y = 0.2398x - 0.9821
\]

\[
R^2 = 0.9534
\]
Notes:

References


Barrington Crater (50,000 years ago), 4000 feet in diameter, approximate size of iron asteroid 45 meters.

October 17, 2012, a car-sized meteor passed northward over the Hayward hills.