Capacitance

Revised: 2010Feb15

Sections 21.7-9 and 23.8 in book

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IV. Capacitance

A. The "Electric Condenser"

B. Dielectrics

C. Energy in Electric Field

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A. The Electric Condenser

1) History of the Capacitor

2) Calculation from Geometry

3) Capacitors in Circuits

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1. History of the Capacitor

a) The Leyden Jar

b) Parallel Plate Capacitor

c) The Law of Capacitance

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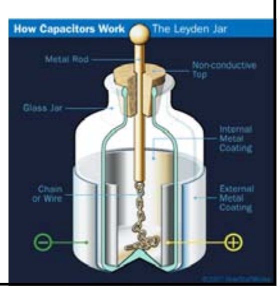
1a. Leyden Jar

Invented in 1745 by Pieter van Musschenbroek (1700–1748) as a device to store "electric fluid" in a bottle



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"Battery" of Leyden Jars

- (1747?) Daniel Gralath was the first to combine several jars in parallel into a "battery" to increase the total possible stored charge.
- He demonstrated its effects on a chain of 20 persons

1b. Aepinus Condenser (c1759)

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The first parallel capacitor (called a “condenser”) was probably developed by Franz Aepinus (around 1759).

The spacing could be changed and materials inserted between the plates.

The Franklin Square

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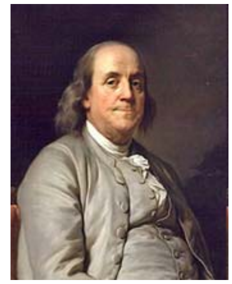
Benjamin Franklin investigated the Leyden jar, and proved that the charge was stored on the glass, not in the water as others had assumed Leyden jars began to be made by coating the inside and outside of jars with metal foil, leaving a space at the mouth to prevent arcing between the foils.

Invented the “Franklin Square”, a capacitor using a square glass plate.



1c. Alessandro Volta’s law of Capacitance

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- 1776 Law of Capacitance:
  Q=VC
- 1782 called the device a “condenser” (derived from the Italian condensatore), with reference to the device's ability to store a higher density of electric charge than a normal isolated conductor

Definition of Capacitance

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- The **C**apacity to store charge
- Formal definition:
  Capacitance=(charge stored)/(voltage applied)
- Units of Capacitance:
  - Farads =Coulomb/Volt
  - Unit is too big! Usually we measure in “microfarads”.

2. Calculation of Capacitance

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Capacitance is a function only of the geometry of the device

a) Parallel Plate Capacitor

b) Cylindrical Capacitor

c) Spherical Capacitors

2a. Parallel Plate Capacitor

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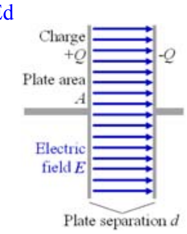
- Voltage between plates: V=Ed
- Electric Field: $E = \frac{Q}{A\varepsilon_0}$
- Capacitance Formula:

$$C = \frac{Q}{V} = \frac{Q}{Ed} = \frac{\varepsilon_0 A}{d}$$



2b. Cylindrical Capacitor

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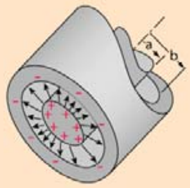
- A coaxial cable is an example of a cylindrical capacitor. The capacitance is given by the formula below where "a" is the radius of inner conductor, "b" is the (inner) radius of the outer conductor and the length is "L".



$$C = \frac{2\pi\varepsilon_0 L}{Ln\left(b/a\right)}$$

2c. Spherical Capacitor

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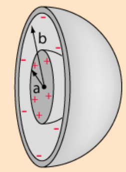
- Faraday experimented with spherical capacitors. The capacitance is given by the formula below where "a" is the radius of inner conductor, "b" is the (inner) radius of the outer conductor.



$$C = \frac{4\pi\varepsilon_0}{\left(\frac{1}{a} - \frac{1}{b}\right)}$$

3. Capacitors in Circuits

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a) Parallel

b) Series

c) RC Circuits

3a. Capacitors in Parallel

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- Capacitors in Parallel add: $C = C_1 + C_2$
- Its like 2 tanks next to each other have more capacity.
- Elements in parallel have same voltage, hence:

$$Q = q_1 + q_2 = C_1 V + C_2 V = (C_1 + C_2)V$$

3b. Capacitors in Series

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- Capacitors in series will have the same charge, but the total voltage is the sum.

$$\frac{1}{C} = \frac{V}{Q} = \frac{V_1 + V_2}{Q} = \frac{1}{C_1} + \frac{1}{C_2}$$

So we have: $$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

Alternate form: $$C = \frac{C_1 C_2}{C_1 + C_2}$$

3c. RC Circuits (incomplete)

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- No notes here, as we did this in detail in lab.
- See lab notes
- See Section 23.8 in Knight (college Physics)
- Time constant for decay
  - Hence Ohm×Farad=Second

$$\tau = RC$$

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B. Dielectrics

1) Dielectric Constant

2) Electric Polarization

3) Dielectric Strength

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1. Dielectric Constant

a. 1837 Faraday finds inserting an insulator (aka "dielectric") between plates will increase capacitance

b. Increase by factor "K" (dielectric constant)
   i. Air: K=1.00059
   ii. Water: K=80

c. The electric field (and hence voltage) is reduced by a factor of "K"

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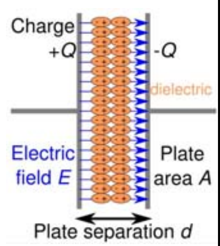
2. Electric Polarization

a. 1833 Faraday shows an electric field "induces" dipoles in an insulator

b. This dipole field opposes the applied field, and so the net field is reduced by a factor of "K"

c. Inside of dielectric, Gauss's law is valid if you replace $\varepsilon_0 \rightarrow \varepsilon = K\varepsilon_0$



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3. Dielectric Strength

a. Definition is the maximum electric field before material ionizes (dielectric breakdown)

b. For Air: 3,000,000 volts/meter

c. This puts a limit on the maximum charge a capacitor can hold.

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C. Energy in Capacitors

1) Energy Storage Formula
2) Electric Stress
3) Energy stored in E field

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1. Energy Storage

a) Analogy: Recall Spring:

- Hooke Force: **F=kx**
- Work: $W = F \bullet \Delta x = k\,x \bullet \Delta x$
- Energy in Spring: $U = \frac{1}{2}kx^2$

Note that it goes like the square of the displacement!

1b. Hydro Analogy

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Energy stored in a tank is proportional to square of the volume!

- Pressure in tank $P = \rho\, g\, x = \rho g\left(\frac{V}{A}\right)$
- Work done $W = P\,\Delta V = \left(\frac{\rho g}{A}\right) V\,\Delta V$
- **Energy in tank:** $U = \frac{1}{2}\frac{\rho g}{A}V^2$

1c. Capacitive

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Energy stored is proportional to square of Voltage (or charge)

$$V = \frac{Q}{C}$$

- Work done $W = V\,\Delta Q = \frac{1}{C}Q\,\Delta Q$
- **Energy in Capacitor:**

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2$$

2. Electric Stress

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a) **Charge on surface of conductor (i.e. plate of capacitor) experiences "pressure" due to the electric field that wants to tear it apart:**

- **Pressure:** $P = \frac{F}{A} = \frac{QE}{A}$
- **Recall from Gauss's law the electric field near a conductor with surface charge:** $E = \frac{Q}{A\varepsilon_0}$
- **Hence:** $P = \varepsilon_0 E^2$

2b. Force Between Plates

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- **The opposite charges on the parallel plates of a capacitor attract each other.**
- **Force must be equal to change in energy:** $F = -\frac{\Delta U}{\Delta x}$
- **There are two ways to argue this. The first is to have a charged capacitor, disconnected from the battery, so that the charge is constant. Then since,** $C = \frac{A\varepsilon_0}{D}$

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\left(\frac{Q^2}{A\varepsilon_0}\right)d$$

$$F = \frac{\Delta U}{\Delta x} = \frac{Q^2}{2A\varepsilon_0}$$

2c. Force Between Plates

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- **We get a different answer however if the capacitor is connected to a battery while the plates are pulled apart.**
- **Here the voltage will remain constant, but the charge will change as the plates are pulled apart (charge will be forced out because the capacitance decreases with increase in distance)**
- **Ouch, need calculus to derive the force between the plates:**

$$U = \frac{1}{2}CV^2 = \frac{\varepsilon_0 A}{2d}V^2$$

$$F = \frac{\Delta U}{\Delta x} = \frac{\varepsilon_0 A}{2d^2}V^2$$

3. Energy Stored in Electric Field

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a) **Energy in terms of Electric Field between plates:**

- **Energy in Capacitor:** $U = \frac{1}{2}CV^2 = \frac{1}{2}VQ$
- **Voltage:** $V = Ed$
- **Relate Charge to Field (in dielectric):** $E = \frac{Q}{A\varepsilon}$
- **Energy=** $U = \frac{1}{2}\varepsilon E^2 (Ad)$

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3b. Energy Density

- **Divide by the volume (Ad) to get the energy per unit volume:** $u = \frac{U}{vol} = \frac{1}{2}\varepsilon E^2$
- **Interpret that the energy IS stored in the electric field**

$$U = \frac{1}{2}\varepsilon E^2 (Ad)$$

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3c. Energy in Dielectric

- By inserting a dielectric the capacitance is increased.
- If capacitor is charged (but not connected to battery), then inserting dielectric will reduce the electric field by a factor of “K”, reducing the energy. Hence it will suck a dielectric into the plates.
- If capacitor is connected to a battery however, then the voltage is constant (E field unchanged) and so energy increases (because of increase in permittivity). Hence will push dielectric out!

$$u = \frac{1}{2}\varepsilon E^2$$

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References

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