Astronomy: Planetary Motion
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Orbital Motion & Gravity
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Orbital Motion
A. Galileo & Free Fall
B. Orbits
C. Newton’s Laws
D. Einstein

A. Galileo & Free Fall
1. Projectile Motion
2. Centripetal Acceleration
3. Galileo & Orbits

1b. Simultaneous Fall
Simultaneous Fall: Galileo shows a bullet fired horizontal will hit ground at same time as bullet dropped.

(why?)

1c. Simultaneous Fall
Both balls fall in the vertical direction at the same acceleration.
Their paths only differ because of the constant horizontal velocity
Galileo proposed that throwing a ball at different speeds causes it to travel farther before it falls to Earth. Throw it fast enough, and as it falls the earth’s curve falls underneath it, and it falls forever (“free fall”).

The critical speed is called **Orbital Velocity**.

For Earth, orbital velocity is 17,500 miles/hr, or 8 km/sec.

2. **Centripetal Acceleration**

- Uniform circular motion: the tangential speed is constant, but the direction of the velocity changes, so there is acceleration towards the center.

\[ a_c = \frac{v^2}{R} \]

\[ v = \omega R \]

\[ a_c = R \omega^2 \]

3. **Orbital Speed**

- Galileo deduces that if the cause of the centripetal acceleration is gravity (centripetal force) then we can calculate the orbital speed

\[ a_c = \frac{v^2}{R} = g \]

\[ v = \sqrt{gR} \]

B. **Orbits**

1. History

2. Kepler’s Laws

3. Newton’s Laws

1a. **Claudius Ptolemy**

- Claudius Ptolemaeu (87-150 A.D.).
- “Geocentric Model” the earth is at the center of the universe
- Planets move on “epicycles”

1b. **Nicolaus Copernicus** (1473-1543 AD)

Common belief was that the earth was the center of the universe, and everything revolved around us.

Copernicus developed the Sun-centered (heliocentric) view of the Universe, which improved the predictions of planetary positions.
1b. Copernican System

- Instead of having 5 deferents with 5 epicycles, you only need 5 circles for the planets.
- The only thing that orbits the earth is the moon.

The only thing that orbits the earth is the moon.

One of the most important books ever... Nicolaus Copernicus
"On the Revolution of Heavenly Spheres" (1543)

The 1000 Zlotych bill features Copernicus. Due to inflation, it was worth about 10 cents USD when I was last in Poland.

2a. Tycho

- Tycho Brahe measuring star positions (without a telescope)
- Measurements of position of Mars showed deviations from Copernican model!
- He built a big observatory with gigantic protractors (no telescopes yet!)

2b. Tycho Brahe’s Uraniborg Observatory

He suggested a weird hybrid model where planets go around sun, but sun goes around earth.
2c. Johannes Kepler (1571-1630)

- Tycho at first invited Kepler to help in analysis of his data, but then jealously wouldn’t let him have the information.
- On his deathbed he gave Kepler the data.
- Kepler used it (particular data on Mars), to develop three laws of planetary motion.

2c.1 Kepler’s 1st Law: Orbits are Ellipses

1605: Kepler realized that the motion of Mars could not be explained with a circular orbit, or the multiple circles proposed by Ptolemy.

- He accepted Copernicus’ view that Mars was in orbit around the Sun, rather than around the Earth.
- He experimented (mathematically) with orbits of various shapes, and found that Mars’ orbit best fits an ellipse.

2c.1 Kepler’s 1st Law (1605)

- Law No. 1. Each planet moves around the Sun in an orbit that is an ellipse, with the Sun at one focus.
  - This is contrary to the earlier belief that the orbits were perfect circles or combinations of circles.

The Ellipse

Do you remember any of this from high school geometry?

- Ellipses, circles (parabolas and hyperbolas) are “conic sections”, studied first by the greeks.
- But it would NEVER occur to the greeks that an orbit is an ellipse. (why?)
Focus Focus
Highly eccentric

Not very eccentric

**TABLE 2.2** Orbital Data for the Planets

<table>
<thead>
<tr>
<th>Planet</th>
<th>Semimajor Axis (AU)</th>
<th>Period (yr)</th>
<th>Eccentricity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mercury</td>
<td>0.39</td>
<td>0.24</td>
<td>0.21</td>
</tr>
<tr>
<td>Venus</td>
<td>0.72</td>
<td>0.62</td>
<td>0.01</td>
</tr>
<tr>
<td>Earth</td>
<td>1.00</td>
<td>1.00</td>
<td>0.02</td>
</tr>
<tr>
<td>Mars</td>
<td>1.52</td>
<td>1.88</td>
<td>0.09</td>
</tr>
<tr>
<td>(Ceres)</td>
<td>2.77</td>
<td>4.80</td>
<td>0.05</td>
</tr>
<tr>
<td>Jupiter</td>
<td>5.20</td>
<td>11.96</td>
<td>0.05</td>
</tr>
<tr>
<td>Saturn</td>
<td>9.54</td>
<td>29.46</td>
<td>0.06</td>
</tr>
<tr>
<td>Uranus</td>
<td>19.19</td>
<td>84.97</td>
<td>0.05</td>
</tr>
<tr>
<td>Neptune</td>
<td>30.06</td>
<td>164.50</td>
<td>0.01</td>
</tr>
<tr>
<td>Pluto</td>
<td>39.60</td>
<td>248.60</td>
<td>0.25</td>
</tr>
</tbody>
</table>

2c.2 **Kepler’s 2nd Law (1609)**

Kepler also noticed that when Mars is closest to the Sun in its elliptical orbit, it moves faster than when it is farther away.

This led him to formulate his Second Law of Planetary Motion.

2c.2 Kepler’s 2nd Law (Equal areas in Equal Times)

According to his second law, a planet moves fastest when closest to the Sun (at perihelion) and slowest when farthest from the Sun (at aphelion). As the planet moves, an imaginary line joining the planet and the Sun sweeps out equal amounts of area (shown as colored wedges in the animation) in equal intervals of time.

2c.2 **Kepler’s 2nd Law**

Kepler’s 2nd law is actually a form of conservation of angular momentum:

\[
\frac{\Delta A}{\Delta t} = \frac{1}{2} r v = \frac{1}{2} m r \frac{v}{m} = \frac{L}{2m}
\]
2c.3 Kepler's 3rd Law: “Harmonic Law”

Planets closer to the sun move faster.
This is consistent with his 2nd law, that showed a planet will move faster at perihelion.
He searched for a relationship between orbital period and distance to the sun.

2c.3 Kepler’s 3rd Law (1618)

- The square of the orbital period \((P)\) is directly proportional to the cube of the semimajor axis of the orbit \((a)\).
  \[ P^2 = a^3 \]
  This law explains the proportions of the sizes of the orbits of the planets and the time that it takes them to make one complete circuit around the Sun.
  [Note: in physics, the symbol “a” is also used to represent "acceleration". Confused?]

Why is it called the "harmonic law"? Kepler thought the spacing between planets was related to musical intervals.

<table>
<thead>
<tr>
<th>Table 2-3</th>
<th>A Demonstration of Kepler’s Third Law</th>
</tr>
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<tbody>
<tr>
<td>Sidejury period (P) (yr)</td>
<td>Semimajor axis (a) (AU)</td>
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An example of Kepler’s third law: The orbit of Mars
(Recall: \(P^2 = a^3\))

- Mars’ orbit period \((P)\) is 1.88 years. \(P^2 = 3.53\)
- Kepler’s law says that \(P^2 = a^3\), so \(3.53 = a^3\).
  So then \(a = (3.53)^{1/3}\) (the cube root of 3.53), or 1.52.
  Thus, the semimajor axis (average distance of Mars from the Sun) is 1.52 Astronomical Units.
  But how big is an Astronomical Unit?
  Kepler didn’t know.

"How large is the Astronomical Unit?"

Astronomical Unit (AU)
The average distance from the Earth to the Sun
150,000,000 kilometers, or 93,000,000 miles

But how was this measured?
**The Newton-Kepler Law**

- In the Principia Newton also deduced Kepler’s third law, but in an important new form
- Mass of central body: \( M = a^3/P^2 \)
  - Orbital Radius “\( a \)” (in astronomical units)
  - Period “\( P \)” (in years)
  - Mass “\( M \)” in units of “solar masses”
- To measure mass of
  - Earth, use moon’s orbit
  - Jupiter, use Galilean moons
  - Sun, use orbits of planets
  - Galaxy, use orbits of stars around galaxy

**In SI units**

- With Newton’s law of gravity, his 2nd law and Galileo’s centripetal acceleration, we can derive:

\[
\frac{m}{R} = \frac{GM}{R^2} \\
v = \frac{2\pi R}{P} \\
M = \frac{v^2R}{G} = \frac{4\pi^2 R^3}{GP^2}
\]

**C. Law of Gravity**

1. Inverse Square Law
2. Newton’s 4th law
3. Acceleration of Gravity

**1a. Inverse Square Law**

- Apparent Luminosity drops off inversely proportional to squared distance.
- Sun at planet Saturn (10\( \times \) further away than earth) would appear 1/100 as bright.
- Sound behaves the same way
- So do electric and magnetic forces

**1b. Inverse Square Law**

- Acceleration of gravity is inversely proportional to distance (from center of earth)
- Example: At the surface of the earth (one earth radii distance) the acceleration of gravity is nearly \( g = 10 \text{ m/s}^2 \)
- The moon is 60\( \times \) further away
- Acceleration of moon towards earth is hence 60\( ^2 \times \) smaller (about \( a = 0.003 \text{ m/s}^2 \))
2. Gravity: Newton’s 4th Law

(a) The apple tree story
"After dinner, the weather being warm, we went into the garden and drank tea, under the shade of some apple trees," wrote Stukeley, in the papers published in 1752 and previously available only to academics.

"He told me, he was just in the same situation, as when formerly, the notion of gravitation came into his mind. It was occasion'd by the fall of an apple, as he sat in contemplative mood. Why should that apple always descend perpendicularly to the ground, thought he to himself."

(b) The Law of Gravitation
• The mutual force between two bodies is proportional to their masses, and inversely proportional to square of distance.
• Newton could not determine the Gravitation Constant “G”

(c) Cavendish Experiment: 1797
Over 100 years later Cavendish measures the constant:

\[ G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2 \]

Very Small! To have 1 N of force would need 1220 kg masses 1 cm apart!

3. The Acceleration of Gravity

(a) Galileo’s Law of Falling Bodies
• Combining Newton’s 2nd and 4th laws, we see that the mass of the test body cancels out!

\[ ma = F = \frac{GmM}{r^2} \]

• Hence we derive Galileo’s law that all test bodies fall at the same acceleration “g”, independent of their mass “m”

3b. Measure Mass of Earth
• Hence if we measure “g”, and know the radius of the earth “r” (measured by ancient greeks), we can determine the mass of the earth!

\[ g = \frac{GM}{r^2} \]

\[ M = \frac{Gr^2}{G} = \frac{\left(9.8 \times \frac{9.8}{6.67 \times 10^{11}} \text{m/s}^2\right)^2}{6.67 \times 10^{-11}} = 6 \times 10^{24} \text{ kg} \]

3c. Escape Speed
• The “gravitational potential energy” is the amount of work we would have to do to lift a mass “m” from surface of earth to infinity.

\[ U = -\frac{GmM}{r} \]

• Equivalently, it’s the amount of Kinetic Energy an meteoroid would have if it fell to the earth.

• Note mass “m” cancels out (all bodies fall at same rate!).

• Hence, there is a minimum “escape speed” such that a body will not fall back to earth!  [about 11 km/sec or 25,000 miles per hour]

\[ \frac{1}{2} mv^2 = \frac{GmM}{r} \]

\[ v = \sqrt{\frac{2GM}{r}} = \sqrt{2gr} \]
**D. Gravity Field**

1. Action at a Distance
2. Gravitational Field
3. Black Holes etc.

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**1. Action at a Distance**

“Action at a Distance” (no touching)

- Huygens criticized: How can one believe that two distant masses attract one another when there is nothing between them? Nothing in Newton's theory explains how one mass can possibly even know the other mass is there.

- "actio in distantia" (action at a distance), no mechanism proposed to transmit gravity

- Newton himself writes: "...that one body may act upon another at a distance through a vacuum without the mediation of anything else, by and through which their action and force may be conveyed from one to another, is to me so great an absurdity that, I believe no man, who has in philosophic matters a competent faculty of thinking, could ever fall into it."

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**2. The field concept**

- 1821 Faraday proposes ideas of "Lines of Force"
- Example: iron filings over a magnetic show field lines

-gravitational Analogy:
  - Earth's mass "M" creates a gravity field "g"
  - Force of field on mass "m" is: \( F = mg \)
  - (i.e., "weight")
  - This eliminates "action at a distance"

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2b. Definition of Mass

There are 3 ways to think about mass

1. Inertial Mass \( F = ma \)
2. Passive Gravitational Mass \( F = mg \)
3. Active Gravitational Mass \( g = \frac{GM}{r^2} \)

The "Weak Equivalence principle" says that inertial mass equals passive gravitational mass

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3a. The Equivalence Principle

- Reference at rest with gravity is indistinguishable to a reference frame which is accelerating upward in gravity free environment.

The apple accelerating downward due to gravity looks the same as an apple at rest in space, with the floor accelerating upward towards it.

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3b. Bending of Starlight

- Newton: Light is NOT affected by gravity
- Einstein: Elevator example shows light must be affected by gravity.
- Predicts starlight will be bent around sun!
- 1919 Measured by Eddington!
3c. Black Hole

If the mass of a star is very big and its size shrinks very small, the escape speed becomes bigger than the speed of light, and not even light can escape!

Any mass is compressed into a size smaller than the “Schwarzschild Radius” $R_s$, it will become a black hole.

This can happen during a supernova explosion, or later by additional mass falling on a neutron star.

Anything that comes closer than the Schwarzschild Radius, will fall in and never escape.

3d. Observing a Black Hole

If black how do we see them? Material shed from another star falls towards black hole. Not all the material falls into the hole. Some is ejected at very high energies out “jets” along the axis of the black hole.

3e. Radio Lobes from galaxy Centaurus A

Black holes at the center of galaxies have a mass of over a billion stars combined!