II. Electric Field

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A. Faraday Lines of Force

B. Electric Field

C. Gauss’ Law

A. Lines of Force

1) Action at a Distance

2) Faraday’s Lines of Force

3) Principle of “Locality”

Problems with Action at a Distance

• How does moon “know” that the earth is there pulling on it?
• How is gravity transmitted?
• Why does it follow inverse square law?
• Does it violate “causality” if instantaneous?
• Coulomb’s law has same issues

1. Sir Isaac Newton (1643-1727)

• Proposes gravity must act instantaneously, regardless of distance (else angular momentum not conserved).
• “actio in distans” (action at a distance), no mechanism proposed to transmit gravity

“...that one body may act upon another at a distance through a vacuum without the mediation of anything else, by and through which their action and force may be conveyed from one to another, is to me so great an absurdity that, I believe no man, who has in philosophic matters a competent faculty of thinking, could ever fall into it.*

2a. Sir Humphry Davy 1778 - 1829

• 1807 Electrolysis, used to separate salts. Founds science of electrochemistry.
• His greatest discovery was Michael Faraday.
• 1813-15 takes Faraday with him on grand tour visiting Ampere and Volta.

*Newton himself writes: “...that one body may act upon another at a distance through a vacuum without the mediation of anything else, by and through which their action and force may be conveyed from one to another, is to me so great an absurdity that, I believe no man, who has in philosophic matters a competent faculty of thinking, could ever fall into it.”

1675 Newton proposes an “ether” to transmit forces between bodies
2b. Michael Faraday 1791 - 1867

- 1821 First proposes ideas of “Lines of Force”
  - Example: iron filings over a magnetic show field lines

2c. Electric Lines of Force

- Electric charges create “electric field lines”
- Field lines start on + charges, end on –
- A plus charge will tend to move along these lines

2d. Other Properties

- Field Lines can’t cross (else physics would not be deterministic, ambiguity which way to go)
- Density of lines is proportional to the “strength” of the force

3. Principle of Locality

- Argues that the field lines have independent reality
- Force fields exist as distortions in the “aether” of space
- Alternative to “action at a distance”, particles Locally interact with force lines
- Ideas rejected by others. He can’t put them into mathematical form.

B. Electric Field

1) Definition of Field
2) Sources of Field
3) Electrodynamics

1a. James Maxwell (1831-1879)

- 1855 essay On Faraday’s Lines of Force, suggests lines are like an imaginary incompressible fluid (obeying hydrodynamic equations)
- 1861 paper On Physical Lines of Force, proposes “real” physical model of vortices for magnetic field
1b. Definition of Field

- Definition: force per unit test charge (i.e. don’t want test charge to affect field)

\[
\vec{E} = \lim_{q \to 0} \frac{\vec{F}}{q}
\]

- Units of Newton/Coul (or Volts/meter)

1c. Analogy to Gravity

- Gravitational Force Field: force per unit test mass

\[
\vec{g} = \lim_{m \to 0} \left( \frac{\vec{F}}{m} \right)
\]

- i.e. its an “acceleration of gravity” field

- Mass is the “charge” of gravity: \( F = mg \)

2. Sources of E Field

(a) Point Charge Source (monopoles)
(b) Dipoles
(c) Field of Dipole (incomplete)

2.a Monopole Sources

- A positive charge is a “source” of electric field. Field radiates outward from a point source

- A negative charge is a “sink” of electric field. Field radiates inward

- Field strength: \( E = kQ/r^2 \)

2.b Dipole Sources

- An “electric dipole” is a “stick” of length “L” with + charge on one end and equal – charge on other.

- Dipole moment: \( p = QL \)

- The vector “p” points along axis from – to + charge

- Units (SI) is C\( \cdot \)m

- Standard in Chemistry is the Debye: \( 1D = 3.336 \times 10^{-30} \text{ C} \cdot \text{m} \)

2.c Field of Dipole

- Derivation will be done on board. Basically you use “superposition” of fields of two monopoles.

- Field of dipole along its axis drops off like the cube of the distance!

\[
E(z) = \frac{kQ}{(z - \frac{1}{2}L)^2} - \frac{kQ}{(z + \frac{1}{2}L)^2} \approx 2k \frac{p}{z^3}
\]
3. Electrodynamics

a) Point Charges

b) Torques on Dipoles

c) Gradient Forces

Faraday Cage (1936)

An external electrical field causes the charges to rearrange which cancels the field inside.

B.3.a Point Charge Electrodynamics

- Force on monopole “test charge” $q$:
  $$\vec{F} = q \vec{E}$$

- Force between dipole “$p$” and monopole “$q$” is hence:
  $$F = q E = \frac{2kpq}{z^3}$$

B.3.b Torque on Dipole

- Torque is defined:
  $$\vec{\tau} = \vec{r} \times \vec{F}$$

- Torque on a dipole in an electric field is:
  $$\vec{\tau} = \vec{p} \times \vec{E}$$

- (derivation in class)

B.3.c Gradient Forces on Dipole

- If field is not constant (has a “gradient”) then there will be a force on a dipole:
  $$F = qE(x+L/2) - qE(x-L/2) = p \frac{\Delta E}{\Delta x}$$

- With calculus, force between dipoles (along a line) can be shown to be:
  $$F = 6k \frac{p_1 p_2}{z^4}$$

C. Gauss’s Law

1) Electric Flux

2) Gauss Law (1813)

3) Applications
1. Electric Flux

(a) Review: “Flux” of fluid would be the total flow of density $\rho$ through a surface area $A$, i.e. the flow rate in kg per second.

$$\frac{\Delta m}{\Delta t} = (\rho v) A$$

For light passing through a window, the flux is the total power (Watts), whereas the “Intensity” has units of Watts/m$^2$.

Power=[Intensity] x Area

(b. Definition: Electric Flux

Electric flux is defined to be the electric field (aka “electric intensity”) times the area it “flows” through

$$\Psi \equiv E \cdot A$$

Units: [\Psi]=volt\cdot meter=N\cdot m^2/C

(c. Lambert’s Law

Lambert’s Law (1760)

Intensity is reduced by cosine of angle of incidence

Flux is the dot product of the electric field vector with the “area vector” (which is “normal” to the surface)

$$\Psi \equiv \vec{E} \cdot \vec{A} = EA \cos \theta$$

Sunlight coming in at a low altitude angle will have its energy spread out over more area.

2. Conservation of Flux

(a. Review: If no sources or sinks of fluid, then flux in=flux out (continuity equation)

$$\rho_1 v_1 A_1 = \rho_2 v_2 A_2$$

Another way of saying this is that the flow lines are continuous or that the “Total flux over a closed surface is zero”

$$\oint \vec{E} \cdot d\vec{A} = 0$$

(b. Source of Flux

The net flux out of a closed surface must be due to a “source” inside.

Consider a point charge at the center, then the electric flux would be:

$$\Psi = EA = \frac{Q}{4\pi \varepsilon_0 r^2} \left(4\pi r^2 \right) = \frac{Q}{\varepsilon_0}$$

Note that the result is INDEPENDENT of radius.

(c. Gauss’s Law

In general: The total electric flux through a closed surface is proportional to the TOTAL enclosed charge

$$\Psi = EA = \frac{Q}{\varepsilon_0}$$

This form is really only useful for very symmetric situations. The more general equation involves vector calculus which is beyond the scope of this course:

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$
3. Applications of Gauss’s Law

(a) Spherical Geometry: If the charge distribution is uniformly spherical, then we have the familiar inverse square law result

\[ E = \frac{\Psi}{A} = \frac{\Psi}{4\pi \frac{r^2}{\varepsilon_0}} = \frac{Q}{4\pi \varepsilon_0 r^2} \]

Note that this is valid even if the charge is spread out over a ball (e.g., the surface of a metal sphere).

It also tells us that the electric field INSIDE a hollow ball of charge must be zero!

b. Cylindrical Geometry

Consider a line of charge (such as charge on a wire). The diameter of the wire really doesn’t matter!

Our “Gaussian Surface” is a cylinder of radius “r”

The flux out the ends does not count because the electric field is parallel to those surfaces

\[ E = \frac{\Psi}{A} = \frac{\Psi}{2\pi r L} = \frac{Q/L}{2\pi \varepsilon_0 r} \]

(c. Plane Geometry

Consider a flat sheet (area “A”) with charge “Q” spread out uniformly.

Our “Gaussian Surface” is box with top and bottom of area “A”

The flux out the ends does not count because the electric field is parallel to those surfaces, hence the height of the box can be anything, which tells us the electric field is INDEPENDENT of the distance from the sheet (constant field).

\[ E = \frac{\Psi}{A} = \frac{Q}{2\varepsilon_0 A} \]

(This result first noted by Laplace 1813)

References

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