Philosophy, Mathematics and Formal Logics

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Preface

What we Hope to Accomplish in this Work

An introductory course in formal or symbolic logic is a subject widely required for philosophy majors at both the graduate and undergraduate levels, and is often considered essential to the pursuit of philosophy. Why this should be so is not entirely clear, especially to the students who take the course.

A typical text first teaches the student to "symbolize" certain artificial statements. Here is a random sample quoted from a few highly regarded works which happen to be on my bookshelf:

- Either Adam is blond and loves Eve, or he is not blond and Eve loves him. –Teller (1989)[94].
- If Holmes has bungled or Watson's on the job, then Moriarty will make a mistake. –Jeffrey (1991)[38].
- Either Sam will come to the party and Max will not, or Sam will not come to the party and Max will enjoy himself. –Mendelson (1987)[59].
- If Jones is ill or Smith is away, then neither will the Argus deal be concluded nor will the directors will declare a dividend unless Robinson comes to his senses and takes matters into his own hands. – Quine (1972)[68].

These are to be symbolized into formulas in two-valued "propositional" or "sentence" logic, depending on whether the author believes in propositions or not. Having done this, the student is to learn to solve problems, typically to determine whether a given argument containing assumptions and a conclusion consisting of statements of this sort, is valid or not, by determining whether the resulting formal argument is valid by means of truth tables, or a formal proof of the conclusion from the assumptions (or a combination of both). A similar though more general method is used to solve such problems in first order logic or "predicate logic". Truth trees are also gradually increasing in popularity.

Having been drilled by constant repetition in these methods, one of two outcomes in, in or experience, all too common:

1. The student satisfies the other requirements, but forgets or rarely applies the methods learned in the introductory course.

2. The student then continues the study of logic and finds herself unprepared and disoriented by the material next presented, which is more or less a formal study of meta-logic or meta-mathematics.

Now some students manage to survive the "just do it and don't ask questions" approach, which is arguably the most efficient, and is also not uncommon in the teaching of high school algebra. But students at the college level, particularly students of philosophy, want to know and understand what the subject is about, what it has to do with philosophy, and what the motivations are for the ideas being introduced, and are uncomfortable in proceeding further until they have got some answers with which they are satisfied.

We believe that they deserve answers, and in this book, we try to provide them. To this end, we discuss the basics in each chapter, and reserve more extended discussion of the motivations and philosophical significance of the ideas introduced, as well as more detailed and exact explanations of the nature of the concepts and the abstract objects which represent them, to optional sections. This gives the reader (or the instructor, as the case may be) some flexibility when it comes to deciding which additional material to study.

We also try to be forthright and candid about the scope and applicability of the material introduced, and not to pretend by carefully selected examples, that it is wider than it is.

Some Philosophical Presuppositions Concerning the Nature of Formal Logic

Formal systems of logic, especially two-valued propositional or Boolean logics, have a close affinity to algebra, in particular to *Boolean* algebras. Experience has shown that Boolean logics, and their extension to first and higher order logics, and other related systems, are helpful when it comes to formalizing and clarifying mathematical concepts and proofs. We also try, especially in the concluding chapter, to explain how all these ideas came about, as well as their range of application, and their limitations. Besides that, we try to show how these ideas developed historically, and their connection with philosophy, which since Plato have been deep and numerous.

Introduction

Logic is a branch of mathematics, and the mathematical structures that arise from it are extensively studied, within the framework of such mathematical disciplines as set theory, topology, category theory and lattice theory. It is also a traditional part of the philosophy curriculum, especially at the graduate level, and it receives some attention in mathematical studies and in computer science as well.

It was long the province of metaphysics and of general philosophy, until as a result of the work of George Boole, it became inseparable from mathematics. The study of mathematics has in turn been important in Western philosophy since its inception, at the time of Pythagoras and Plato, and has remained so to this day.

Yet the way in which formal logic is introduced to the beginner is all to often devoid of context and entirely unmotivated. The emphasis is exclusively on the development of "skills", such as symbolizing statements nobody in their right mind would ever make, or on endless drills in performing algorithms. And this does a disservice to the student and to the subject, and makes further progress all the more difficult.

Philosophers do not take things on faith. When they are pursuing a subject, they want to understand what they are doing and why they are doing it. We believe that a basic knowledge of science and of mathematics in particular, is important to philosophy, and to the understanding of reality. You can't philosophize about science without knowing what you're talking about.

We have tried to introduce the subject of logic with candor and without evasion, to explain the underlying motivations and the context which made them important, and hopefully, to give some idea of the manifold possibilities and the diversity of the subject, which like many other fields, has increased dramatically in the last century or so.

In the first introductory chapter, we provide an account of the concepts on which formal logic is based. In the second chapter, we introduce the simple arithmetical operations which define what is meant in symbolic logic by 'not', 'and', and 'or'. In the next chapter, we take a closer look at what this all means than is customary in regular algebra, particularly when it concerns the substitution of actual numbers for the variables in algebraic formulas.

In Chapter 4, we discuss various methods of determining whether a sentence follows from given assumptions, and we also show that these methods really work. In the next chapter, we explore various alternatives to the regular two-valued logic which was heretofore our main concern. In the next three chapters, we extend the discussion to cover generalities and statements of existence, which cannot be done in the logic of "not's, and's and or's" alone. In the final chapter, we provide an account of the philosophical issues which lend importance to the most dramatic discoveries in the past century.

We may add that we take formulas to denote the Boolean truth values 0 and 1, much as in ordinary algebra, formulas denote numbers. We also define formulas involving quantifiers so as not to allow colliding quantifiers, an unnecessary and avoidable subtlety. For similar reasons, we permit only closed formulas in proofs or proof trees. Moreover, we take every element in the domain of a structure to have a name, even if there are uncountably many of them, instead of taking free variables to have variable denotations. If this precludes a physicalist interpretation of proper names, so be it.

We have added optional sections to supplement the core material, hopefully to provide further insight into it, or else alternative approaches. Chapters 5 and 8 are also largely optional. This will, we hope, provide the reader (or instructor) with more flexibility in what subjects to address.

Contents

List of Figures

List of Tables