SANTA CLARA UNIVERSITY — Department of Physics

Is Gauge Invariance Violated by Spin and Torsion?

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- Gauge Violation when add Torsion to EM
- Gauge Dependent EM Field Spin Tensor
- Spin Geo-Electrodynamics and Gauge Problems

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Talk Fri 2004Mar26 3:10-3:45 pm

I. Classical Gauge Invariance

Electrodynamics with Spin and dipoles,

$$\dot{p}_\sigma \;= j^\mu F_{\mu\sigma} + \tfrac{1}{2}\partial_\sigma(\mathcal{U}^{\mu\nu} F_{\mu\nu})$$

$$\dot{S}_{\mu\nu} = (\mathcal{U} \otimes F)_{\mu\nu} - (j \wedge A)_{\mu\nu}$$

$$F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu$$

Field Unchanged by Gauge Transformation

$$A_\nu \longrightarrow A_\nu + \nabla_\nu \theta$$

Invariance of Lagrangian under Gauge Transformation leads to charge conservation

$$\mathcal{L}_I = j^\mu A_\mu + \tfrac{1}{2}\mathcal{U}^{\mu\nu} F_{\mu\nu}$$

$$\delta\mathcal{L}_I = j^\sigma \partial_\sigma \delta\theta = \partial_\sigma(j^\sigma \delta\theta) - \delta\theta \partial_\sigma j^\sigma$$

$$\boxed{(\delta\mathcal{L}_I = 0) \quad \Rightarrow \quad \partial_\sigma j^\sigma = 0}$$

II. Torsion Violates Gauge Inv.

A. Does Torsion contribute to EM Field?

Is the comma to semicolon rule valid?

$$\begin{aligned} F_{\mu\nu} &= \nabla_\mu A_\nu - \nabla_\nu A_\mu \\ &= \partial_\mu A_\nu - \partial_\nu A_\mu - \tau^\sigma_{\mu\nu} A_\sigma \\ &= \mathcal{F}_{\mu\nu} \qquad\qquad\qquad - \tau^\sigma_{\mu\nu} A_\sigma \end{aligned}$$

B. Gauge Violating Force/Torque

$$\begin{aligned} \dot{p}_\mu &= j^\sigma \mathcal{F}_{\sigma\mu} \quad - j^\sigma \tau^\kappa_{\sigma\mu} A_\kappa + \ldots \\ \dot{S}_{\mu\nu} &= (\mathcal{U} \otimes \mathcal{F})_{\mu\nu} - \mathcal{U}_{[\mu}{}^{\sigma}\, \tau^\kappa_{|\sigma|\nu]} A_\kappa + \ldots \end{aligned}$$

C. How Restore Gauge Invariance?

- Denial: There is no torsion

  (a)However, Cartan's equations imply,

  Spin $\Longleftrightarrow$ Torsion

  (b) Don't Photons follow autoparallels?

- Discrimination: Photon can selectively decides to couple to curvature but NOT torsion

III. Momentum Conservation

Particle stress-energy is transferred to field

$$\nabla_\mu T^{\mu\nu}_{\text{pl}} = j_\mu F^{\mu\nu} = -\nabla_\mu T^{\mu\nu}_{\text{em}}$$

Derivations of EM Stress Tensor:

1. Gravitational: $$T^{\mu\nu} = \frac{\delta \mathcal{L}}{\delta g_{\mu\nu}}$$

2. Noether's: $$T^{\mu\nu} = \frac{\delta \mathcal{L}}{\delta \partial_\mu A_\sigma} \partial^\nu A_\sigma - g^{\mu\nu} \mathcal{L}$$

- Asymmetric, unconserved angular momentum
- Differs from (1) by Gauge Dependent term

3. From Maxwell Equations (Riesz 1947)

(a) $$\Box F = j \qquad \begin{cases} \Box \cdot F = j \\ \Box \wedge F = 0 \end{cases}$$

(b) $$F \Box = -j$$

Multiply (b) on right by F, (a) on left by F, add

$$-2j \cdot F = F \overleftrightarrow{\Box} F = \partial_\mu (F \mathbf{e}^\mu F)$$

Vector part yields correct (symmetric) tensor

$$\boxed{T^{\mu\nu} = \mathbf{e}^\nu \cdot (F \mathbf{e}^\mu F) = F^{\mu\sigma} F_\sigma{}^\nu - \tfrac{1}{4} g^{\mu\nu} F^{\sigma\kappa} F_{\sigma\kappa}}$$

Great Question: Does a plane wave carry spin?

• Feynman (Lect Phys 17-10, 1965) argues circularly polarized light transfers spin to a plate.

• Recent Debate argues instead its edge-effect angular momentum on finite sized target
AJP **69**, (2001), 405, **70**, (2002), 565 & 567.

• Another author has proposed a point-like classical EM spin tensor (physics/0102084)

• But the idea has met with resistance

Rejected by: Phys. Rev. D (25 Sep 2001), Foundation of Physics (28 May 2001), American J. of Physics (15 Sep 1999, 10 Sep 2001, 28 Mar 2002), Acta Physica Polonica B (28 Jan 2002, 09 May 2002), Phys. Lett. A (22 July 2002), Experimental & Theor. Phys. Lett. (14 May 1998, 17 June 2002), J. Experimental & Theor. Phys. (27 Jan 1999, 25 Feb 1999, 13 Apr 2000, 25 May 2000, 16 May 2001, 26 Nov 2001), Theor. Math. Phys. (29 Apr 1999, 17 Feb 2000, 29 May 2000, 18 Oct 2000), Physics - Uspekhi (25 Feb 1999, 12 Jan 2000, 31 May 2000), Rus. Phys. J. (18 May 1999, 15 Oct 1999, 1 March 2000, 25 May 2000, 31 May 2001, 24 Nov 2001). **Rejected by the arXiv** (21 Jan 2002, 18 Feb 2002, 02 June 2002, 13 June 2002).

IV. Local Spin Conservation?

(NOT origin-dependent angular momentum)
Is particle Spin transferred to Field?

$$\nabla_\sigma S_{\text{pl}}^{\sigma[\mu\nu]} = (\mathcal{U} \otimes F)^{\mu\nu} = -\nabla_\sigma S_{\text{em}}^{\sigma[\mu\nu]}$$

1. Does EM Field Have Spin?

Consider infinite circular polarized beam

- No angular momentum
- Does it have spin (which causes torsion?)
- Will it cause a plate to rotate?

2. If it Has Spin

- Cartan: spin $\Rightarrow$ torsion
- Should it "feel" torsion?
- Since Torsion is Gauge Violating ...

3. Spin Tensor is Gauge Dependent?

- Ad hoc[1]: $S_\sigma{}^{[\mu\nu]} = A^{[\mu}\partial_\sigma A^{\nu]}$
- Gives $S^{412} = \pm 1$ for circular polarized
- Conservation: $\nabla_\sigma S^{\sigma\mu\nu} = (A \wedge j)^{\mu\nu}$

V. Derivation of Spin Tensor

Much easier with Clifford Algebra

$$(1) \quad \Box F = j + \Box \mathcal{U} \qquad \begin{cases} \nabla_\mu (F + \mathcal{U})^{\mu\nu} = j^\nu \\ \nabla_{[\sigma} F_{\mu\nu]} + \nabla_{[\sigma} \mathcal{U}_{\mu\nu]} = 0 \end{cases}$$

$$(2) \quad F + \mathcal{U} = \Box \wedge A = -A \Box, \qquad \text{where } \Box \cdot A = 0$$

Multiply (1) on left by A, (2) on right by F, add to get multivector conservation law,

$$A \overset{\leftrightarrow}{\Box} F = A(j - \Box \mathcal{U}) - (F + \mathcal{U})F$$

Take Bivector part,

$$\nabla_\sigma S^{\sigma[\mu\nu]} = -(\mathcal{U} \otimes F)^{\mu\nu} - (j \wedge A)^{\mu\nu}$$
$$+ A \wedge \Box \cdot \mathcal{U} + A \cdot \Box \wedge \mathcal{U}$$

Gauge Dependent Spin Tensor

$$S^{\sigma[\mu\nu]} = A^\sigma F^{\mu\nu} + 2A^{[\nu} F^{\mu]\sigma} + 2g^{\sigma[\mu} F^{\nu]\kappa} A_\kappa$$

Gives $S^{412} = \pm 1$ for circular polarized

$$\mathcal{S}^3 = S^{412} = A^4 B^3 + (\vec{A} \times \vec{E})^3$$

VI. EM Lagrangian + Torsion

$$\mathcal{L} = \sqrt{-g}\,(\tfrac{1}{4}\,F^{\mu\nu}F_{\mu\nu} + j^{\mu}A_{\mu} + \tfrac{1}{2}\,\mathcal{U}^{\mu\nu}F_{\mu\nu})$$

Separate out Gauge Dependent Torsion terms

$$F_{\mu\nu} = \mathcal{F}_{\mu\nu} - \tau^{\sigma}_{\mu\nu}A_{\sigma} \ ,$$

$$\mathcal{F}_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu} \ .$$

Regroup terms of free part,

$$\mathcal{L} = \tfrac{1}{4}\,\mathcal{F}^{\mu\nu}\mathcal{F}_{\mu\nu} - \tfrac{1}{2}\,\tau^{\sigma}_{\mu\nu}\,A_{\sigma}\mathcal{F}^{\mu\nu} + \tfrac{1}{2}\tau^{\sigma}_{\mu\nu}A_{\sigma}\,\tau^{\kappa\,\alpha\beta}A_{\kappa}$$

Interpret first-order gauge dependent term as torsion coupling to EM spin?

$$\tau^{\sigma}_{\mu\nu}A_{\sigma}\mathcal{F}^{\mu\nu} = \quad \tau^{\sigma}_{\mu\nu}S^{\nu\mu}{}_{\sigma} + \tau^{\sigma}_{\sigma\nu}S_{\kappa}{}^{\kappa\nu}$$

- If no EM spin, can ignore torsion
- If no torsion, can ignore EM spin
- Gauge violation issues of torsion and spin are related!

VII. Spin Source of Gravity

Bailey[9] spin contribution to stress energy,

$$T^{\mu\nu} = T_0^{\mu\nu} + \nabla_\sigma S^{\mu\sigma\nu}$$

suggests (ad hoc) modified Lagrangian,

$$\mathcal{L} = \tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} + g_{\mu\nu} \nabla_\sigma S^{\mu[\sigma\nu]}$$

Spin part of action is a **gauge-dependent** boundary term:

$$\mathcal{Q}_s = \int d^4x \, g_{\mu\nu} \nabla_\sigma S^{\mu[\sigma\nu]} \;\cong\; \oint d^3\Sigma_\sigma \, A_\kappa F^{\sigma\kappa}$$

If demand $\delta\mathcal{Q} = 0$ under $\delta A_\kappa = \nabla_\kappa \delta\theta$

$$\int d^4x \, \delta\,\theta \, \nabla_\mu j^\mu = \oint d^3\Sigma_\sigma \, (\delta\theta j^\sigma + F^{\sigma\kappa} \nabla_\kappa \delta\theta)$$

To insure charge conservation:

- on boundary: $\delta\theta = 0$
- on boundary: $\nabla_\kappa \delta\theta = \delta A_\kappa = 0$

Hence adding a constant voltage to the universe is not allowed!

VIII. Spin Geo-Electrodynamics

Particles follow paths (polygeodesics) which extremize the length swept out by the momentum $\pm$ the area swept out by the spin.[7,8]

$$L = m_o \sqrt{\overset{\circ}{x}{}^{\mu}\,\overset{\circ}{x}{}_{\mu} - \frac{1}{2\lambda^2}\,\overset{\circ}{a}{}^{\mu\nu}\,\overset{\circ}{a}{}_{\mu\nu}} \qquad \overset{\circ}{Q} \equiv \frac{dQ}{d\kappa}$$

- New Affine Parameter (based on Invariant)

$$d\kappa^2 = dx^{\mu} dx_{\mu} - \frac{1}{2\lambda^2} da^{\mu\nu} da_{\mu\nu}$$

- Fundamental length λ (radius of gyration)
- Yields Papapetrou Equations, with torsion

Interaction $\quad L_I = e\left(\overset{\circ}{x}{}^{\mu} A_{\mu} - \frac{1}{2}\,\overset{\circ}{a}{}^{\mu\nu} F_{\mu\nu}\right)$

Curvature (even without torsion) induces **gauge dependent** equations of motion

$$\overset{\circ}{p}{}_{\sigma} = j^{\nu} F_{\nu\sigma} + \tfrac{1}{2}\mathcal{U}^{\mu\nu}\left(\nabla_{\sigma} F_{\mu\nu} + R_{\mu\nu\sigma}{}^{\omega} A_{\omega}\right)$$

$$\frac{1}{\lambda^2}\left(\overset{\circ}{S} - \mathcal{U} \otimes F\right)_{\mu\nu} \simeq \tfrac{1}{2}\left(\mathcal{U} \otimes F + j \wedge A\right)^{\sigma\omega} R_{\mu\nu\sigma\omega} + \left(j^{\alpha}\Gamma^{\omega}_{\alpha\mu} + \tfrac{1}{2}\mathcal{U}^{\alpha\beta} R_{\alpha\beta\mu}{}^{\omega}\right) F_{\omega\nu}$$

IX. Summary

Conjecture: Gauge Dependent electrodynamic force is "balanced" by Gauge-Dependent Spin contribution to EM stress

$$\nabla_\mu T^{\mu\nu}_{\text{pl}} = -\nabla_\mu \left(T^{\mu\nu}_{\text{em}} + \nabla_\sigma S^{\mu[\sigma\nu]}_{\text{em}} \right)$$

System as a whole has Gauge Invariance, but only if classical spin of EM field is included.

- **Gauge-Dependent Torques?** Must we include "spin" carried off by the gravitational field?

- **Torsion & Spin Electrodynamics:** Messier, but suspect the Gauge-Dependence can be "repaired" the same way.

X. References

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