

*Is **E=mc²** wrong?*

or

is something missing?

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I. Introduction

II. Something Missing?

III. Is Everything Relative?

IV. Is $E=mc^2$ wrong?

V. Summary

I. Introduction

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A. Outline of Talk

B. MetaPhysics (about Physics)

C. Light: The Starry Messenger

World Year of Physics 2005

Einstein in the 21st Century



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I B2. MetaThemes

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How do I talk about Einstein's theory without mathematics?

Fortunately, the principles of physics are not mathematical in origin.



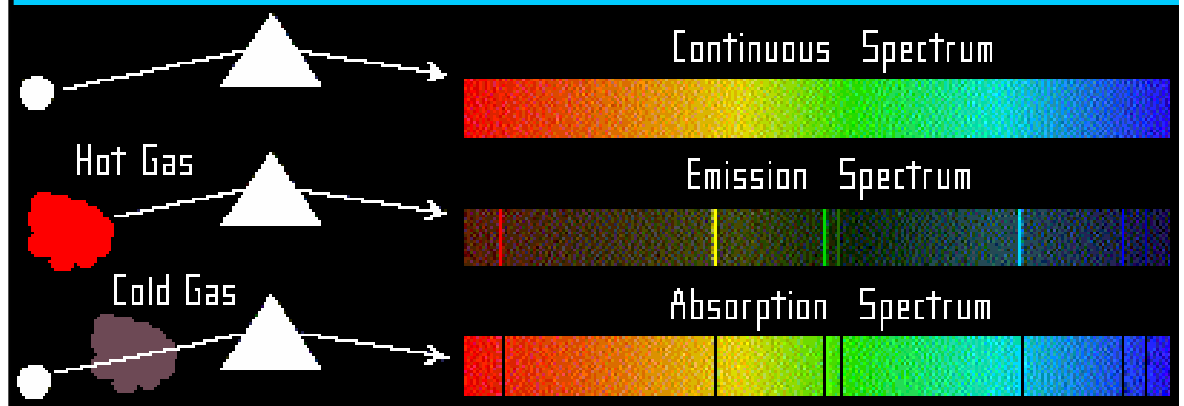


**All we know
about the universe
comes from light**

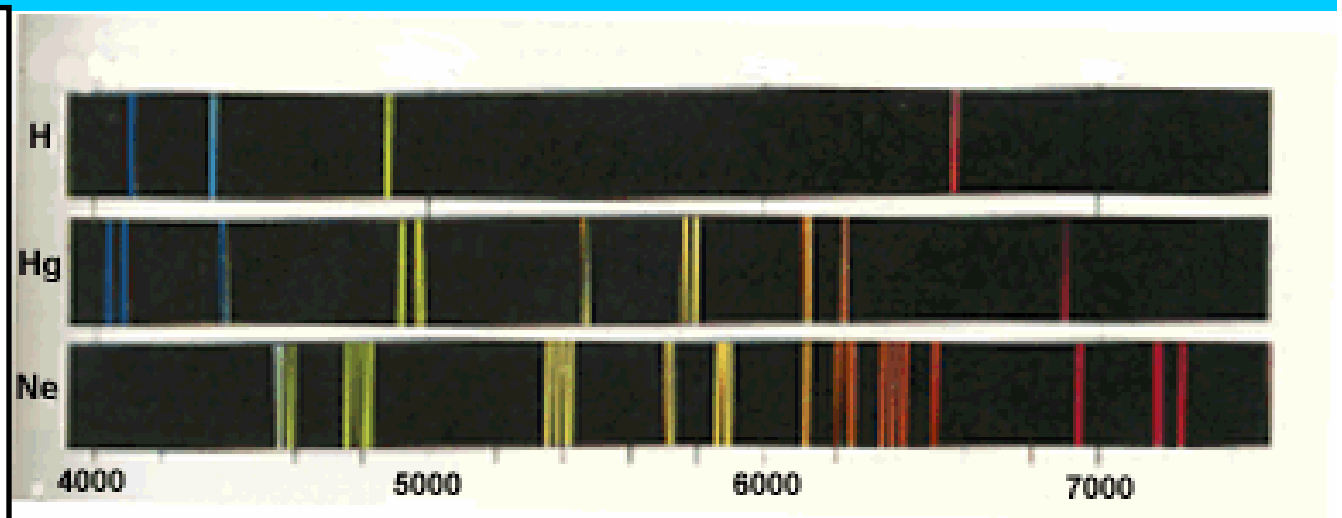
I C2. Review: Spectra

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•If you spread starlight out with a prism, you find “spectral lines” contributed by the atoms.



•Each element has unique lines in a precise location (wavelength)



I C3. Review: Doppler Effect

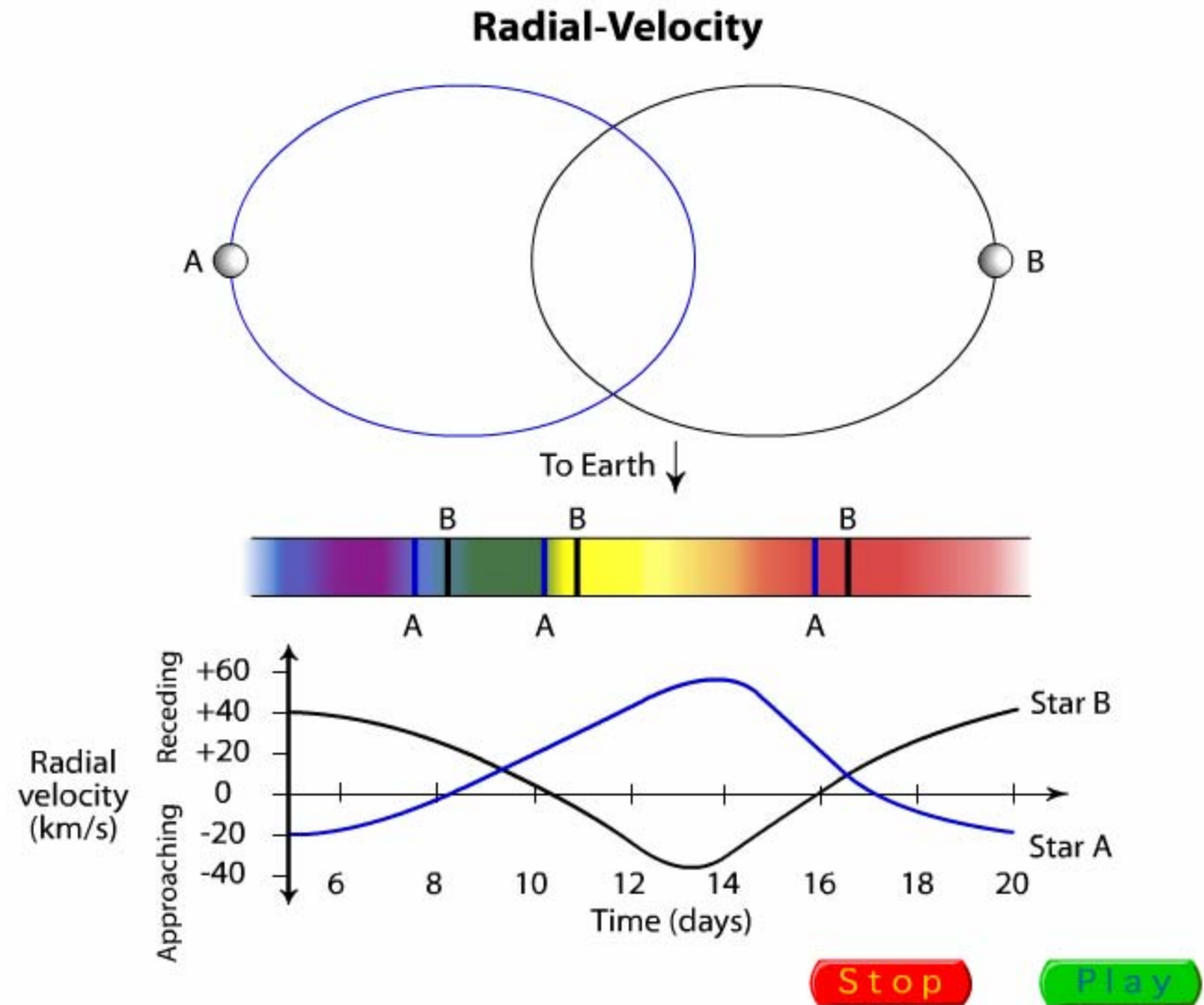
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Example:
Binary Star

Redshift:

when moving away from us, the spectral lines are shifted to the red.

Shift is proportional to speed



II. Is Something Missing?

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A. Missing (Dark) Matter

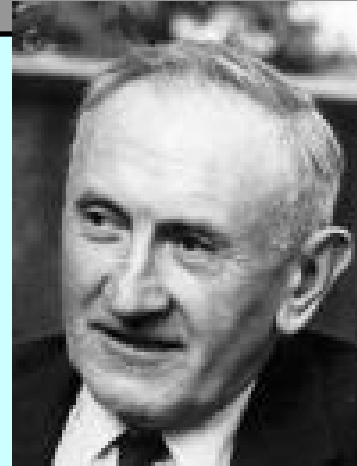
**B. Einstein's worst (best)
mistake & Dark Energy**

C. Pioneer 10 Anomaly

A1. Dark Matter

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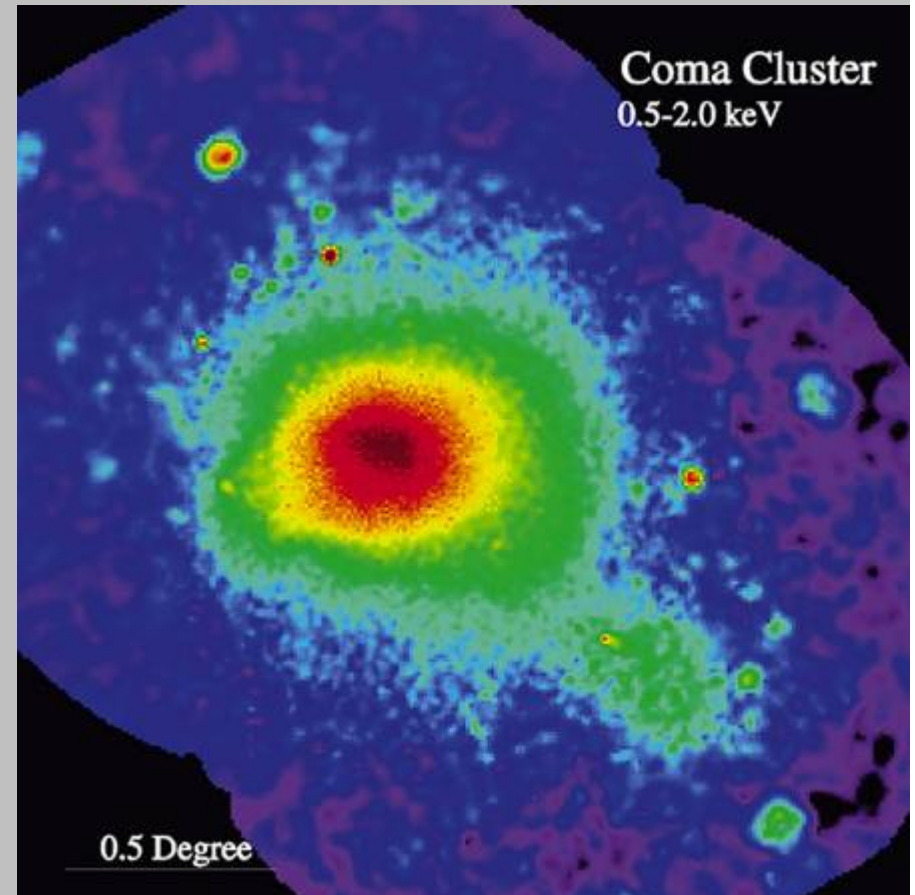
- 1933 Fritz Zwicky
- Galaxies orbiting Coma Cluster moving much faster than could be explained by mass of cluster
- Luminous Mass is only 10% of total
- Other 90% mass is “missing”
- Coined term “**Dark Matter**”



A1b. Dark Matter in Clusters

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- 1970s Ultra Hot “X-ray gas” filling the space between the galaxies in a cluster
- Hot gas would have very high velocity, and would escape unless there is sufficient gravitational mass keeping it there.
- The mass of all the visible galaxies is only 10% of the required amount to explain the trapped gas.
- Again 90% must be **Dark Matter**



- <http://astronomy.nmsu.edu/tharriso/ast110/class25.html>

A2. Rotations of Galaxies

- Spiral Galaxies Rotate Slowly
- Sun takes 226 million years to go around (220 km/sec or 1 AU in 8 days)

•The rotation speed can be measured by the Doppler effect

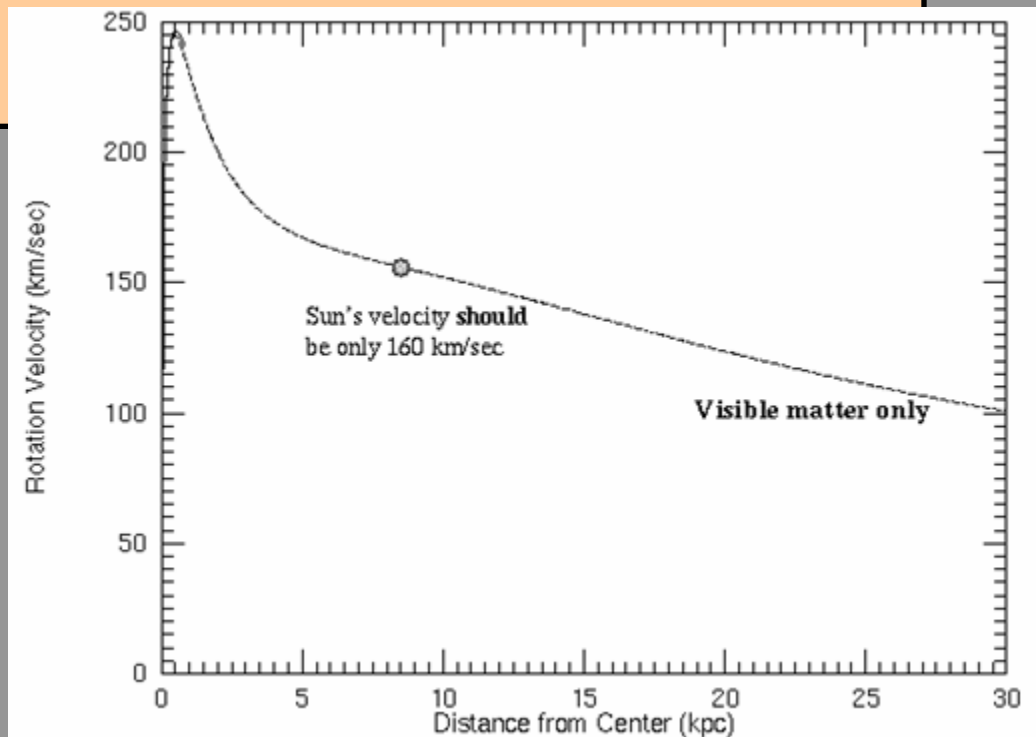
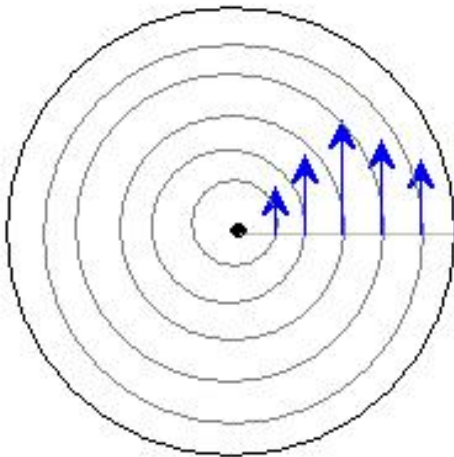


A3. Rotation Curves

- Assuming most of mass of galaxy is in the core
- Velocity of a Star is related to distance from core:

$$V^2/R = GM/R^2$$

• Or: $V \propto 1/\sqrt{R}$



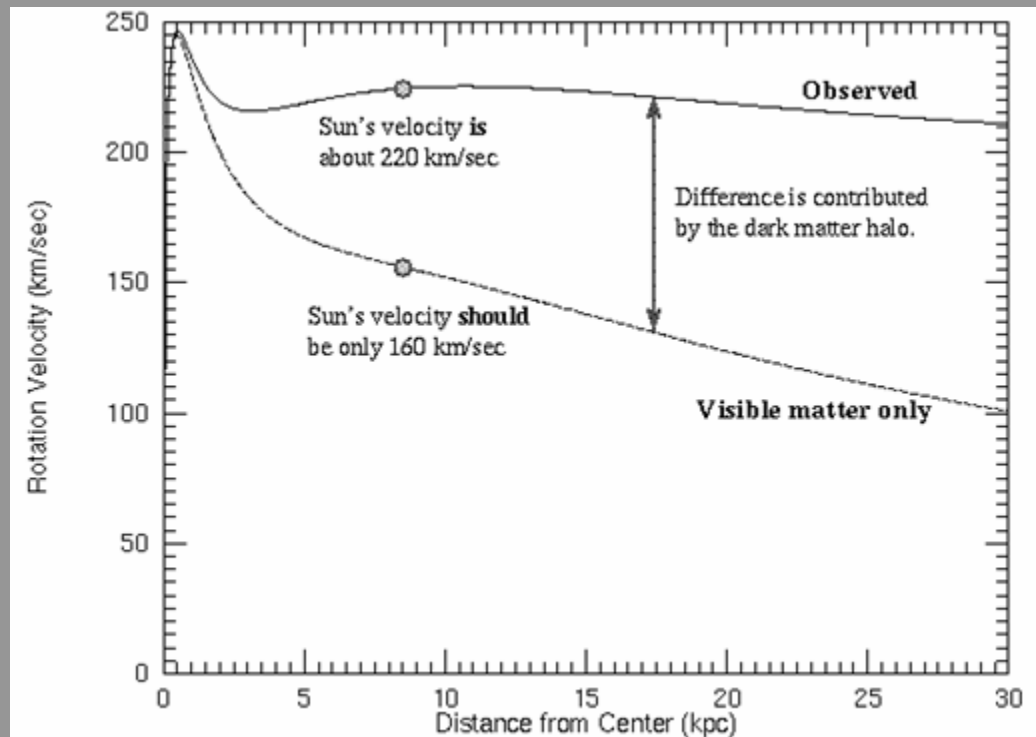
A4. Rotating Rong?

14



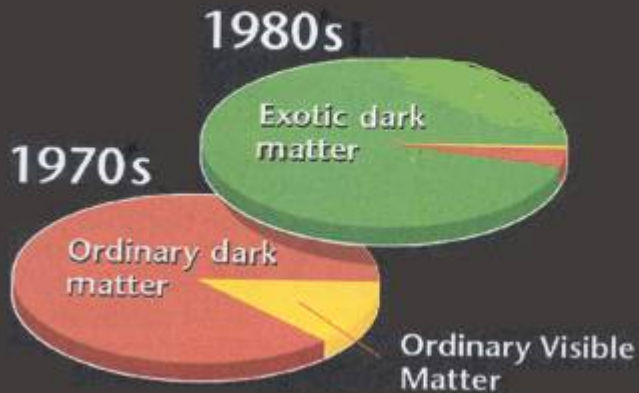
- 1980 Vera Rubin shows rotation curves of galaxies are nearly constant!
- Implies a lot of “missing” (dark) matter surrounds galaxies.

Pivotal Paper:
Rotational Properties of 21 Sc Galaxies with a Large Range of Luminosities and Radii from NGC 4605 (R=4kpc) to UGC 2885 (R=122kpc), *Astrophys. J.* 238: 471 (1980),
V.C. Rubin, W. K. Ford, Jr. and N. Thonnard.



A5. What IS Dark Matter?

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- MACHOs (Massive Compact Halo Objects) were looked for:
 - White Dwarfs
 - Brown Dwarfs
 - Black Holes
- But its not enough!

•WIMPs (Weakly Interacting Massive Particles):
Must propose exotic things like a neutrino, but with BIG mass (10 to 10,000x that of proton).

Even though 96% of the universe is made of it, not a single piece of it is in this room.

•Or maybe there is something wrong with our theory of gravity?

B1. *Static Cosmology*

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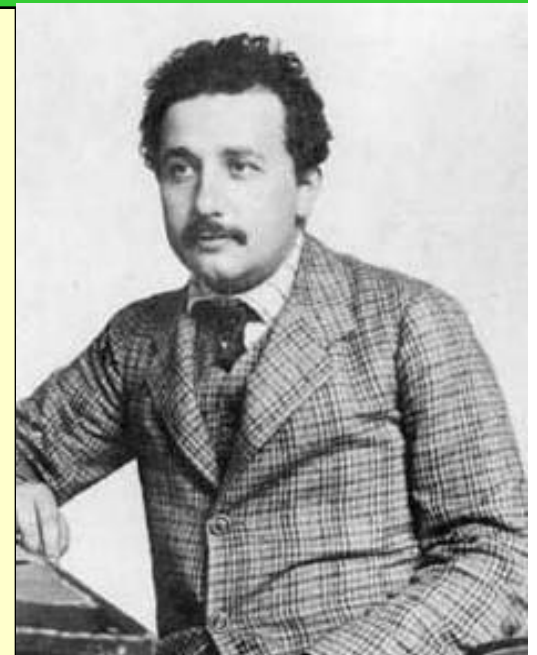
Newton proposed that the universe must be infinite to be balanced; a finite universe would collapse due to gravity



- 1920 Eddington shows that Newton's infinite universe is unstable and would collapse

1917 Einstein proposes universe:

- Is finite, curved like a ball
- But gravity would still collapse it
- Proposes negative pressure (cosmological constant) prevents collapse
- Later calls this his "biggest blunder"

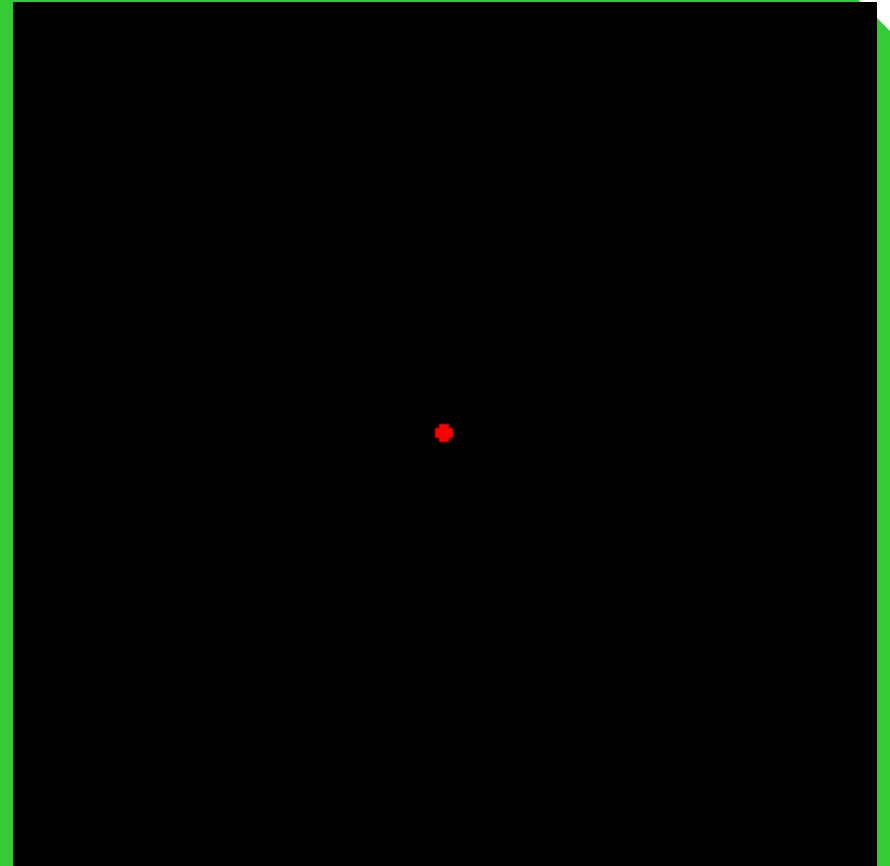




B2. The Expanding Universe

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•1922 Friedmann shows that another solution would be that the universe is expanding from kinetic energy leftover from a “big bang” creation. *No need for negative pressure.*

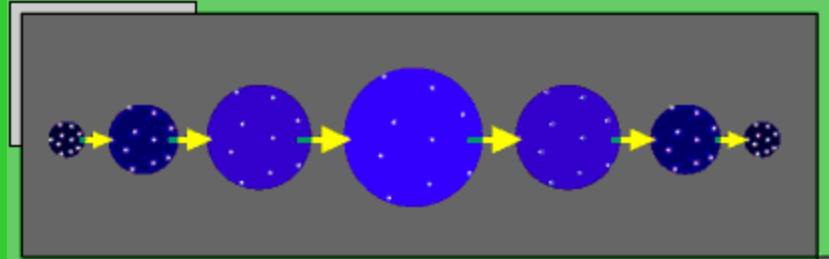


•1929 Hubble verifies that the universe is indeed expanding.

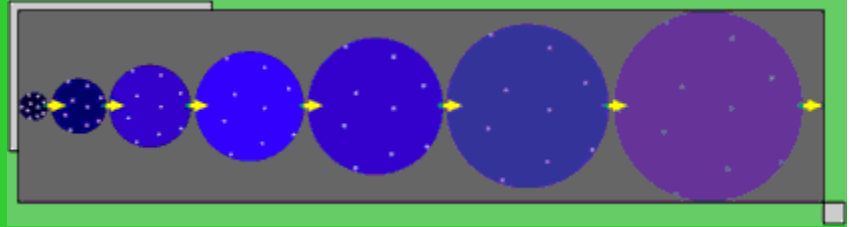
B3. Decelerating Universe

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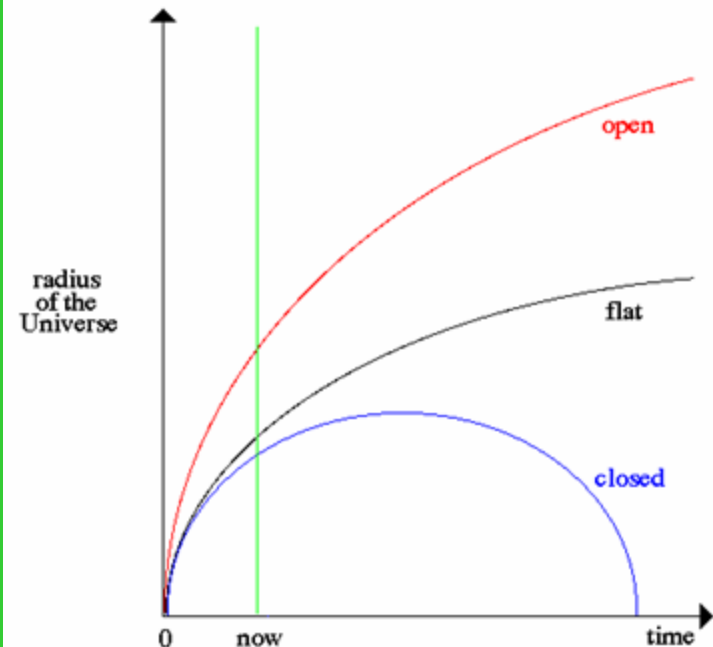
CLOSED UNIVERSE: Just like a ball thrown upward will fall back to earth due to gravity, we might expect the universe will slow down, and collapse.



OPEN UNIVERSE: If you throw a ball upward fast enough, it won't fall back, but it certainly will slow down due to gravity



Everyone assumed that the universe must be decelerating due to gravity, it was only a question of how fast it was slowing down.



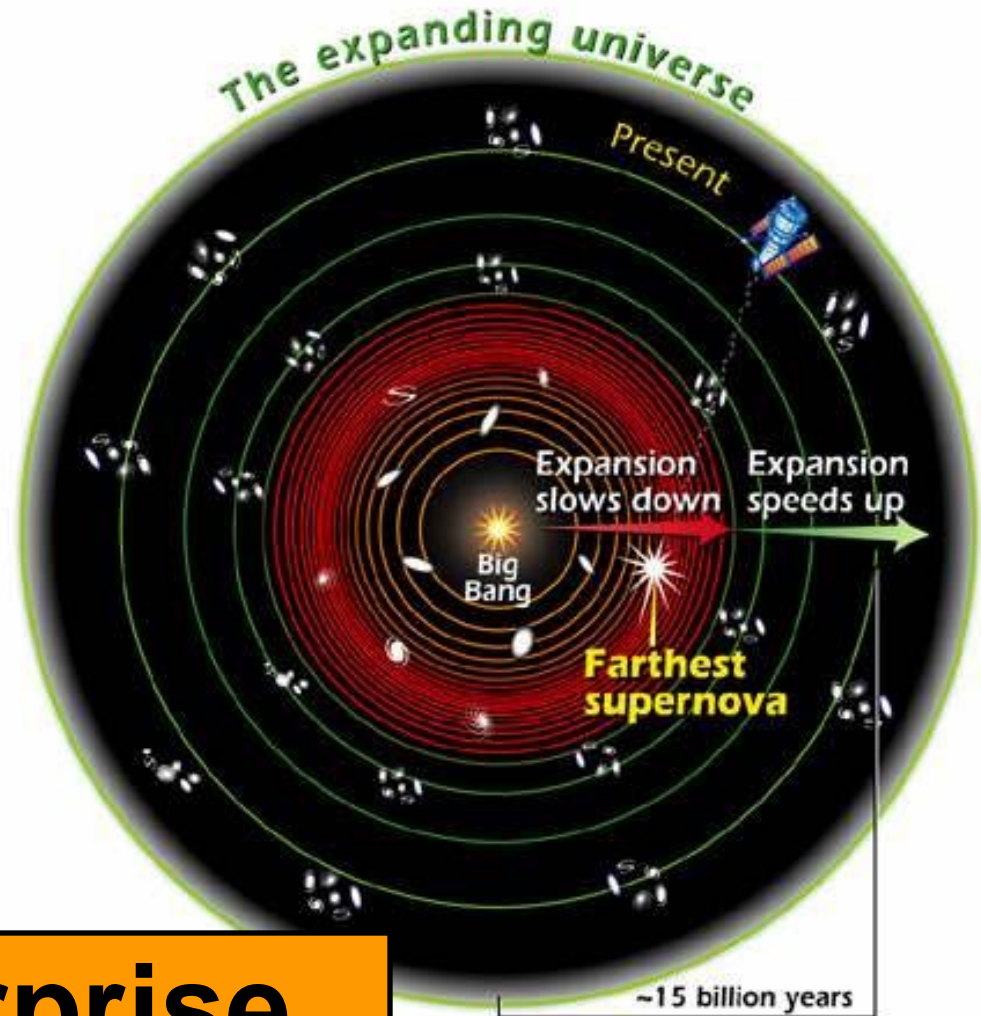
B4. The Universe is Accelerating?

19

1998 Measurements of distant supernova (i.e. in the distant past) were 20% fainter than expected.

Interpretation:
universe WAS
slowing down for first
half of lifetime

BUT, since then, it
has been
ACCELERATING



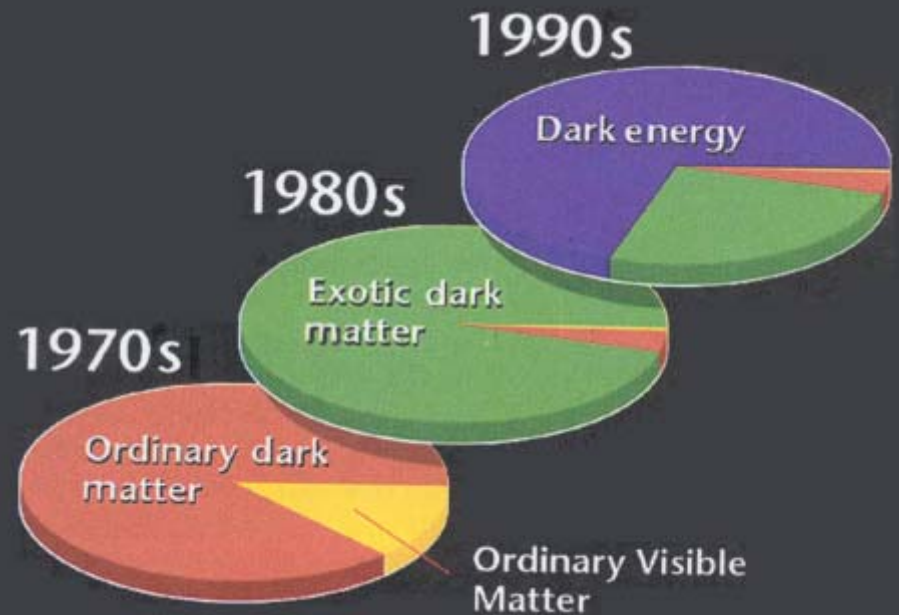
This is a big surprise

B5. Dark Energy

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How do we explain this?

- Propose Universe is
 - 4% normal stuff
 - 23% weird dark matter
 - 73% “DARK ENERGY”



The Dark Energy provides the negative pressure.

But is the universe really 96% weird stuff?

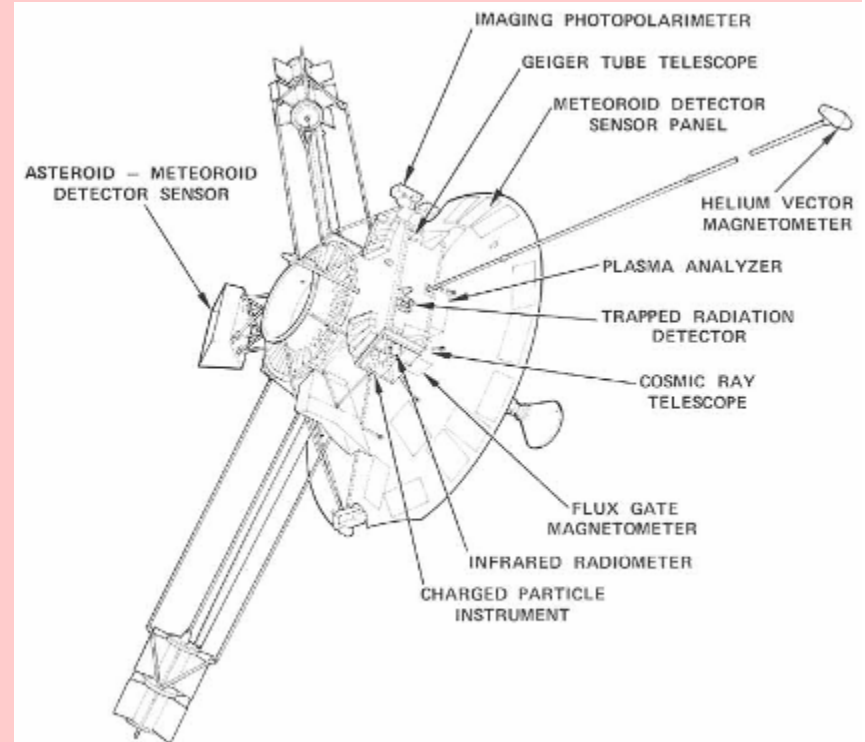
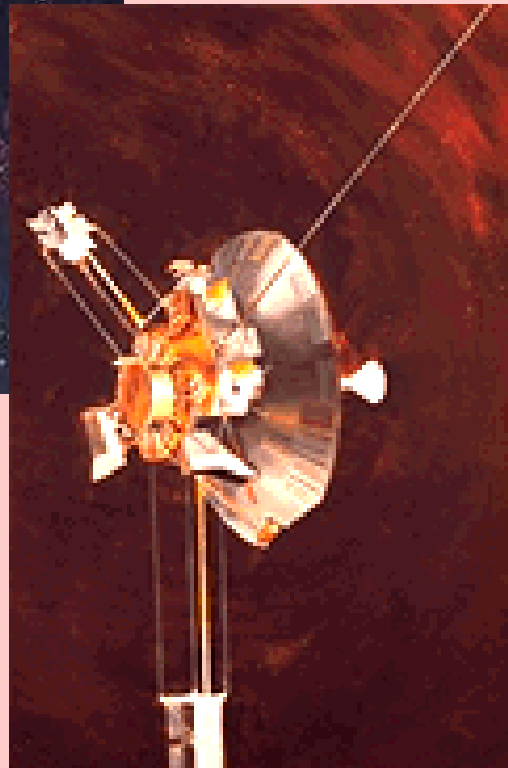
- Alternative: Revive Einstein’s Cosmological Constant?

C1. Pioneer 10/11 Anomaly

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Launched: 1972/1973
Mission: Jupiter & Saturn

http://spaceprojects.arc.nasa.gov/Space_Projects/pioneer/PNhome.html



C2. Pioneer 10/11

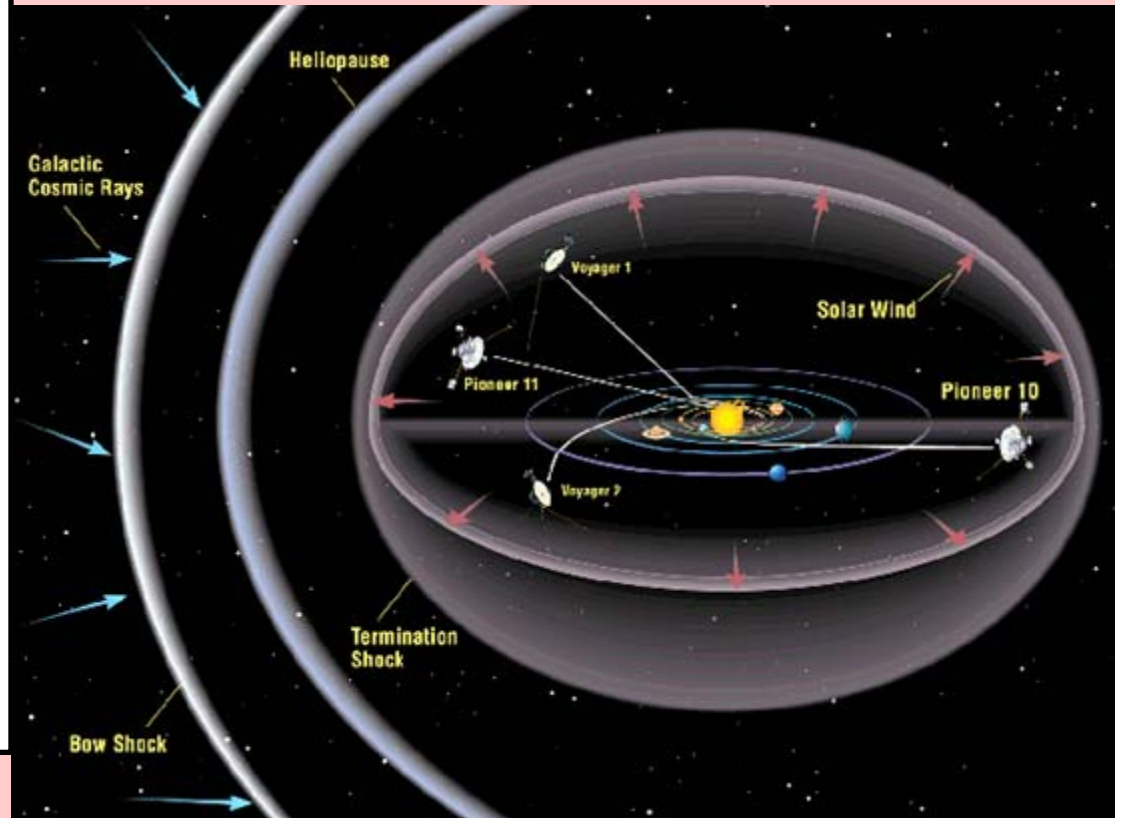
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- 1983 Pioneer 10 at 30 AU, passes Neptune/Pluto, First spacecraft to leave solar system!
- 1998 Pioneer 10 at 70 AU, traveling 2.5 AU/year

1980 John Anderson,
principle gravitation
investigator, notes
**anomalous acceleration
toward sun**

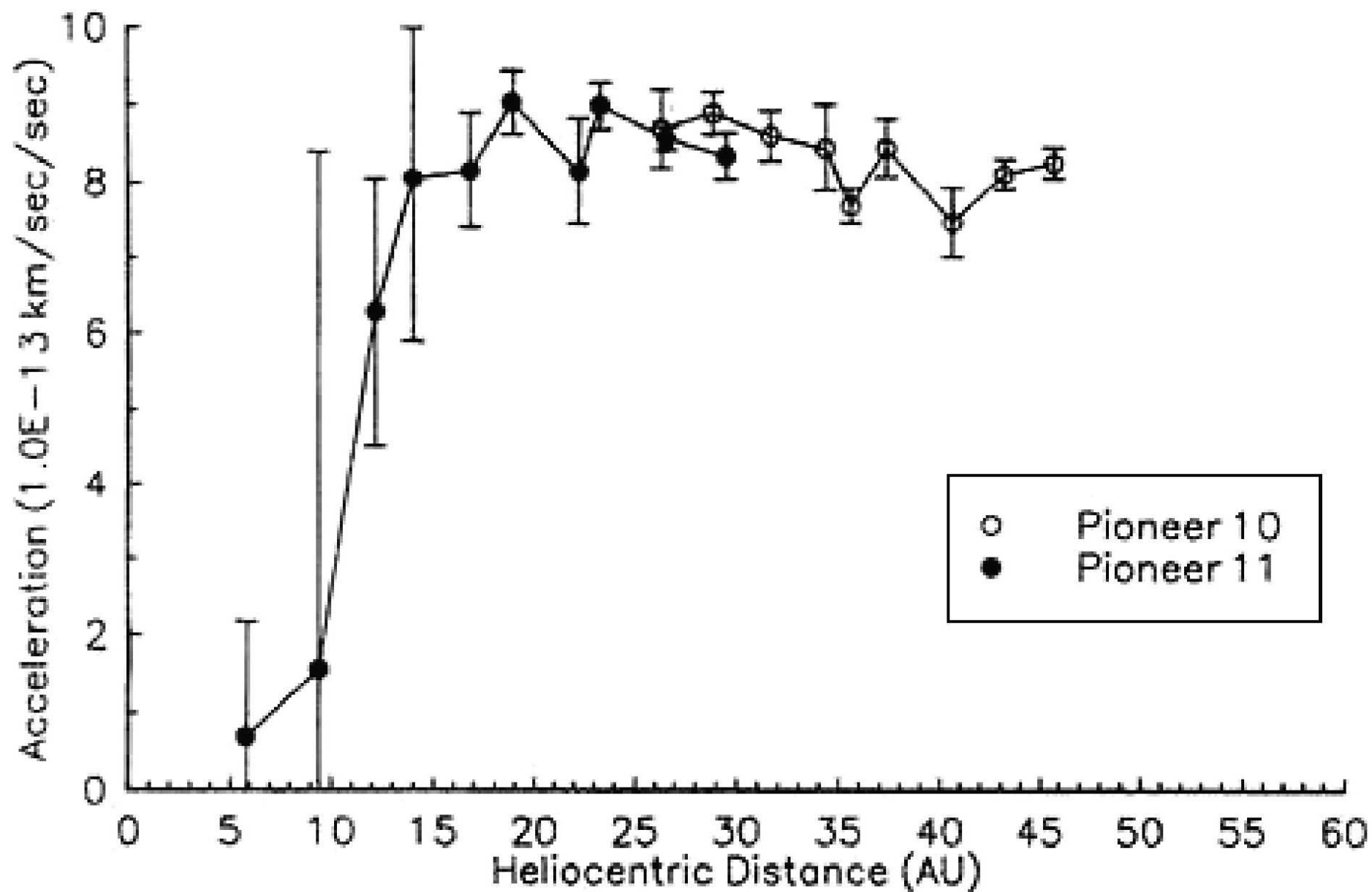
1995 NASA funds grant to
study

2002 Paper Published



UNMODELED ACCELERATIONS ON PIONEER 10 AND 11

Acceleration Directed Toward the Sun



C4. How big is this?

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Consider Pioneer reached Neptune (30 AU) after 10 years

At 30 AU	Effect
Sun's Gravity	$-1.5 \times 10^{-5} \text{ m/s}^2$
Solar Wind	$< + 2 \times 10^{-10} \text{ m/s}^2$
Anomalous	$-8.74 \times 10^{-10} \text{ m/s}^2$
Change in range in 10 years due to effect	-0.003 AU

•Turyshchev et al, AJP 73, 1033 (2005), "Study of the Pioneer anomaly: A problem set"

C5. How Explain it?

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Why does Pioneer 10/11 see effect, but planets don't? Possibilities:

- **Moving away from sun
(while planets circle sun)**
- **Spin is parallel to motion
(helicity-spin-gravity effect?)**
- **Doppler equation is wrong?
(rotational doppler effect?).**

Part II: Summary

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Is the universe really 96% weird stuff
(Dark Matter, Dark Energy),

but none of it has ever fallen to the earth?

OR

**Is there something wrong with the
theory of gravity?**

III. Is Everything Relative?

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A. Mach and Rotational Relativity

B. Rotating Frame Paradoxes

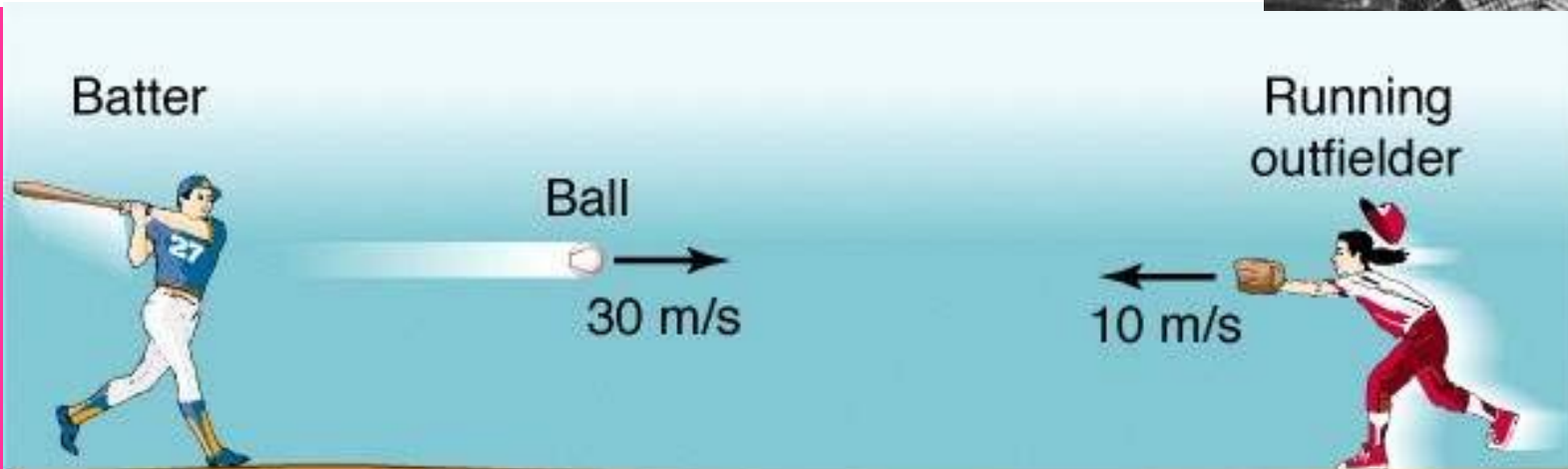
C. Field Rotation & Torsion

A1. Special Relativity

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1905 Einstein (26 years old) publishes theory of special relativity

- Speed of light is the same for all observers
- Motion is relative (Galileo)
 - there is no experiment one can do to determine absolute motion relative to “space”.



As seen by outfielder, ball is approaching her at $(30 \text{ m/s}) + (10 \text{ m/s}) = 40 \text{ m/s}$

A2. Rotational Relativity

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Newton argued that water in a rotating bucket will make the shape of a parabola due to centrifugal force.

The presence of centrifugal and coriolis forces confirm that you are in a non-inertial rotating frame of reference





A3. *Mach's Principle*

30

1883 Mach argued that one can not tell if the bucket is rotating, or instead

The stars
are
rotating
around
the
bucket



A4. Frame Dragging

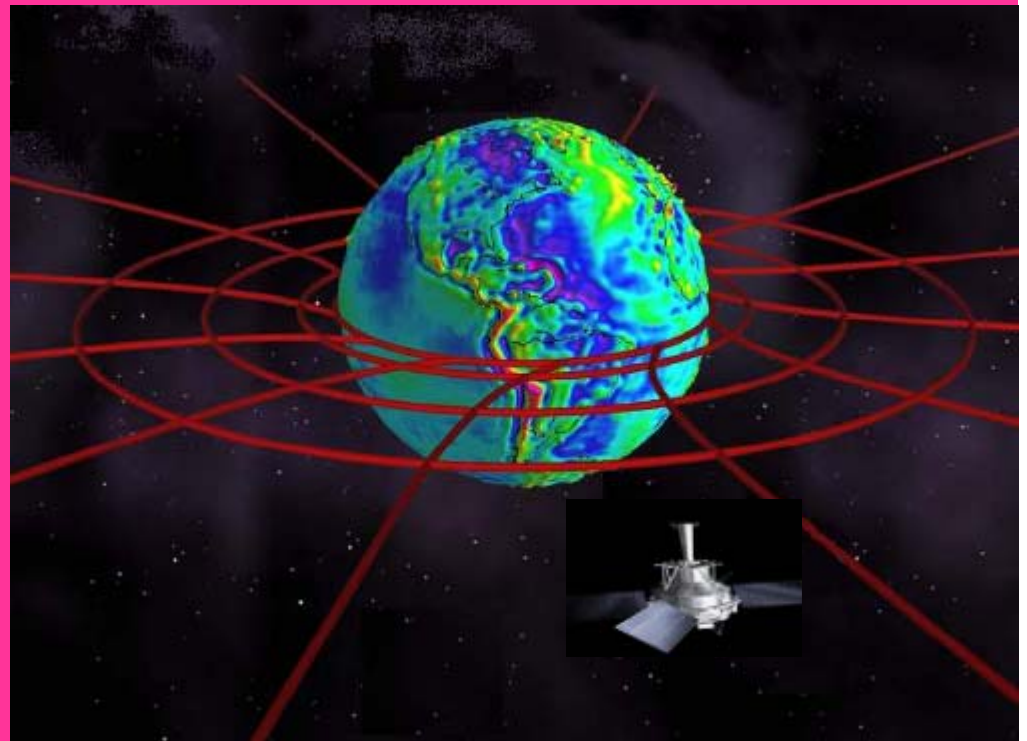
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Mach argued a centrifugal force will appear in both cases, indicating only a relative rotational motion

1896 Friedlander attempted (failed) to measure if there is a centrifugal force introduced inside of a big rotating flywheel.

Frame Dragging:

Recently Gravity Probe B has been orbiting the earth to see if there is a similar effect: that the rotating earth pulls the space around with it



A5. Machian vs Gravitomagnetic

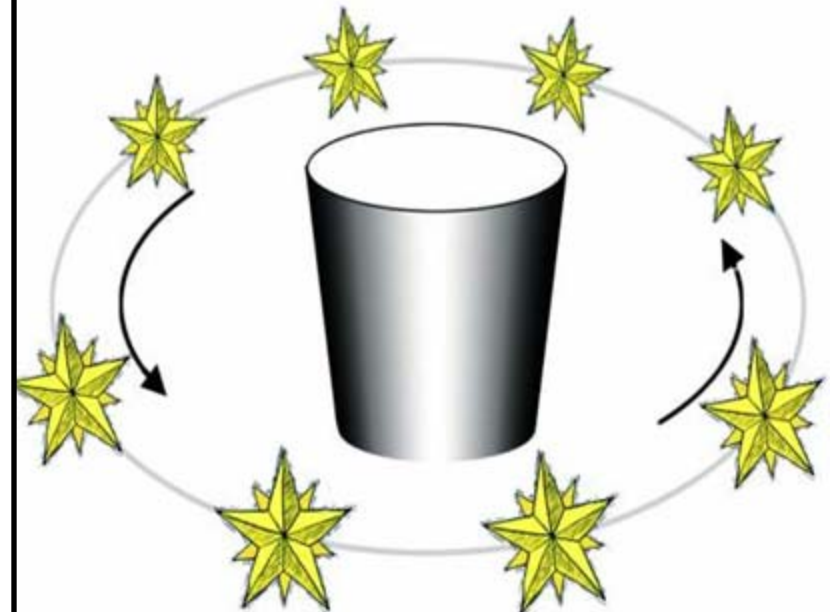
32

Newton's gravity is sort of the "electric" part of GR
Frame dragging is sort of a "magnetic" part of GR

If there is 10x more mass rotating, the frame-dragging gravitomagnetic effect should be 10x bigger.

The Machian centrifugal force however should be the same if there is 8 stars out there rotating about the bucket, or a billion

They are not the same!



B1. Lorentz-FitzGerald Contraction

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1889 FitzGerald, 1892 Lorentz

- **Propose a moving meter stick will appear to shrink in length**

$$L' = L \sqrt{1 - \left(\frac{v}{c}\right)^2}$$

- **1905 Einstein deduces this from his postulates of relativity.**

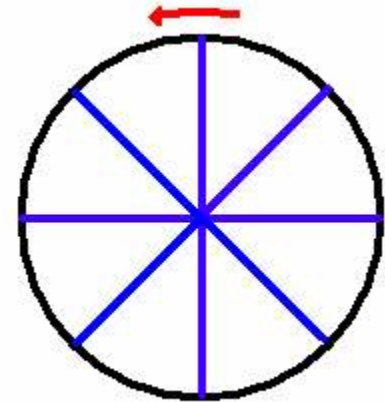
B2. Ehrenfest Paradox 1909

34

Ehrenfest proposes puzzle about a rotating disk.

• FRAME OF DISK

- Radius “R”
- Circumference $C = 2\pi R$



• FRAME OF Lab

- Rotating Disk's Radius will be unchanged (motion is perpendicular to radius)
- Circumference is moving so is it shrunk?
 $C' = C [1 - v^2/c^2]^{1/2}$

How can a rotating circle have a circumference smaller than $2\pi R$?

B3. Sagnac Effect (1913)

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Send laser beams around earth to other side. (67 mns)

Lab Frame: because of rotation of earth, eastward beam has to go further, takes longer.

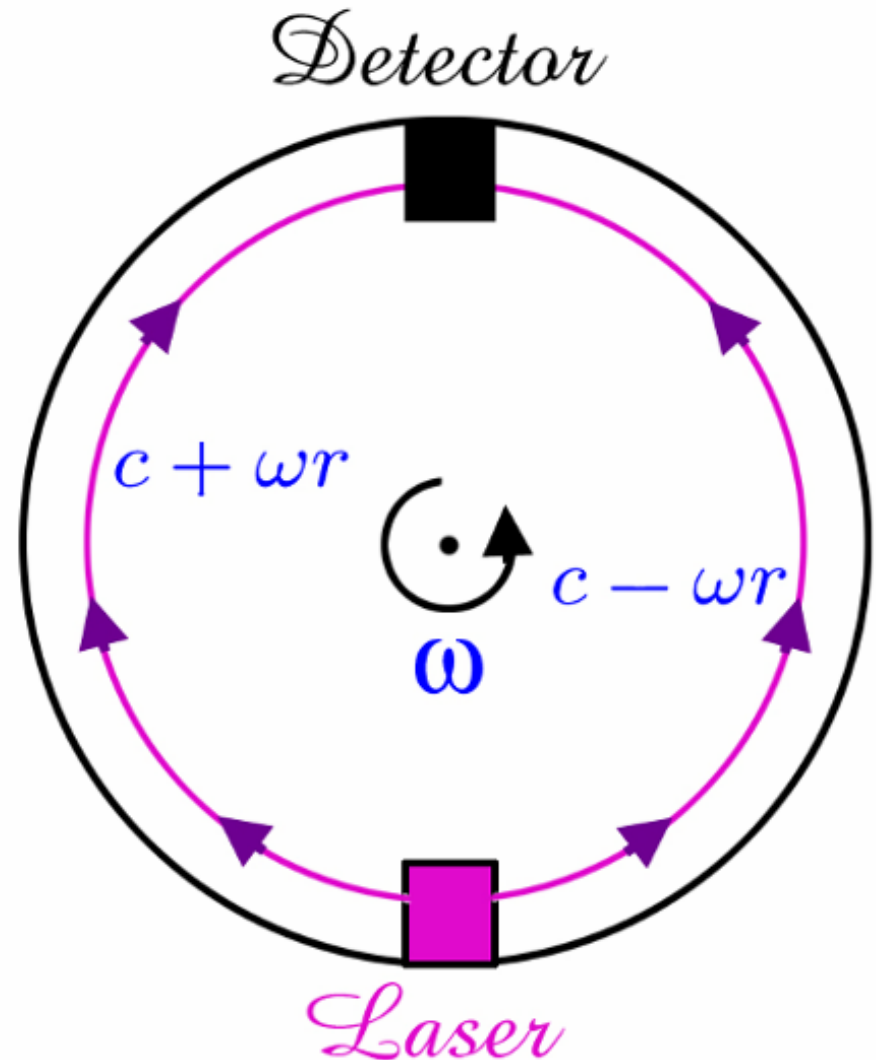
$$T_{\pm} = \frac{r\pi}{c \mp \omega r}$$

Rotating Frame:

• Speed of light appears to change with direction $(c \pm \omega r)$

• Time gap measured (0.22 usec)

$$\Delta T = \frac{2\omega(\pi r^2)}{c^2 - \omega^2 r^2}$$



B4. Absolute Rotation?

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Universal Nature of Sagnac Effect

- Independent of physical nature of beams
- Problem synchronizing clocks on earth
- Time asynchronization must somehow be induced by setting frame into rotation

Interpretation(?)

- **There is NO special relativity of rotation**
- **Sagnac effect is a measure of absolute rotation**

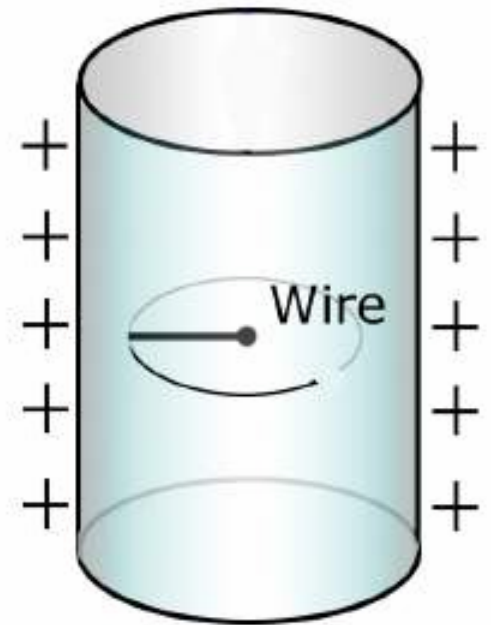
C1. Barnett's Experiment (1912)

37

There are analogies to Mach's problem for electromagnetic fields in rotating frames.

Consider charged cylinder at rest

- Inside the electromagnetic fields are zero.
- A wire (at rest) inside will have no voltage across it.



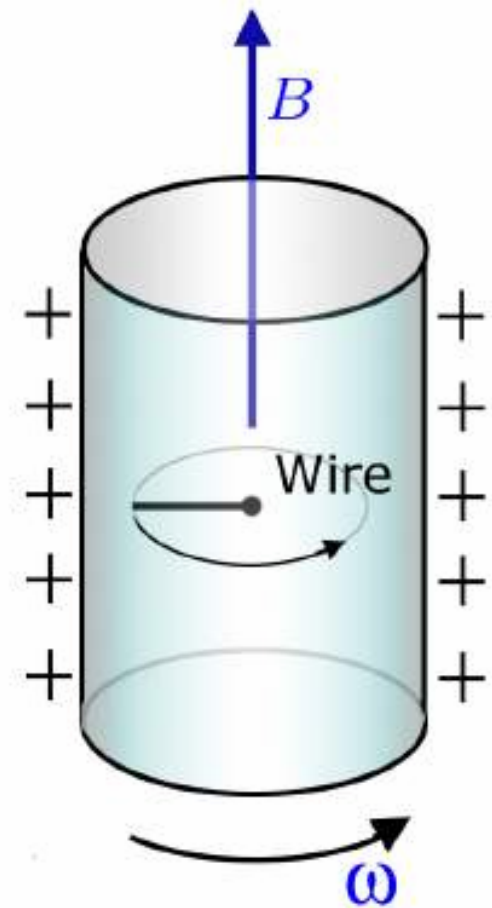
C2. Barnett's Experiment (1912)

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Put the cylinder into rotation

Lab Frame

- Magnetic field induced
- Wire moving through magnetic field will have a $(\mathbf{v} \times \mathbf{B})$ Lorentz force on it
- A voltage is generated across the wire

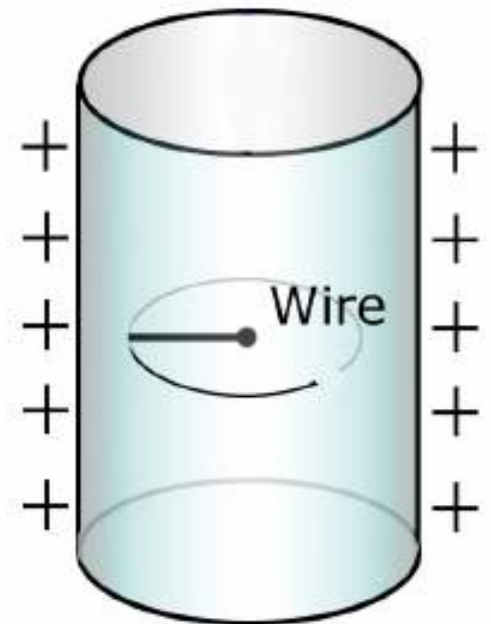


C3. Barnett's Experiment (1912)

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Rotating Frame

- When you do the experiment, an electric field is found inside (consistent with Lorentz transformation of magnetic field)
- Hence there is voltage across the wire
- Rotational Relativity would demand Gauss' Law to hold, hence NO electric field. Hence NO voltage measured. **Contradiction!**



C4. Field Rotation Paradox

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Standard View:

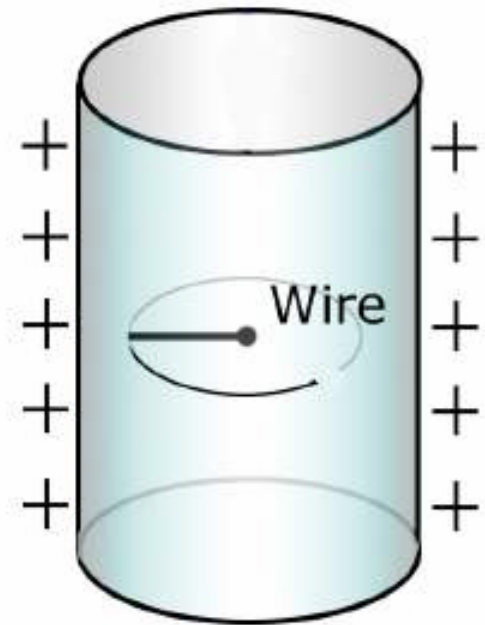
- there is NO theory of special rotational relativity.
- The voltage across the wire is a measure of absolute rotation
- But, absolute relative to what? Empty space?

Non-Standard View:

- Corum (1980) shows rotating electrodynamics is consistent with a frame at rest including **TORSION**

- Torsion is a “twisty” nature to space, which Einstein excluded in his theory of gravitation (he only included “curvature”)

- In a space with torsion, a static electric charge will create a magnetic field!



C5. Curvature

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Einstein's General Theory of Relativity

Gravity is “curved space”

Big curvature makes a **Black Hole** that you can fall in and never get out

For example, when people throw things into my tuba, they are never seen again.



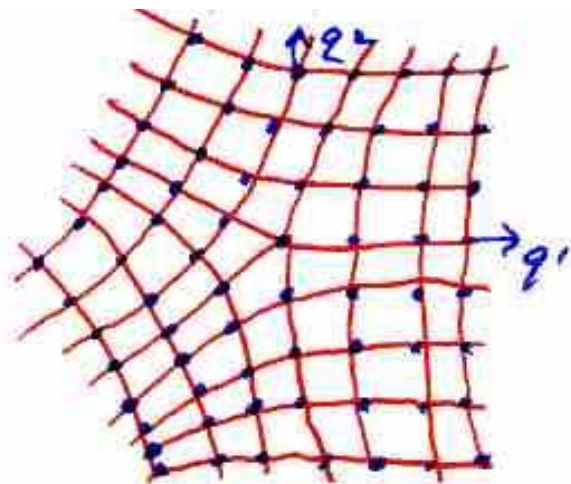
C6. Torsion (Teleparallel Theory)

42

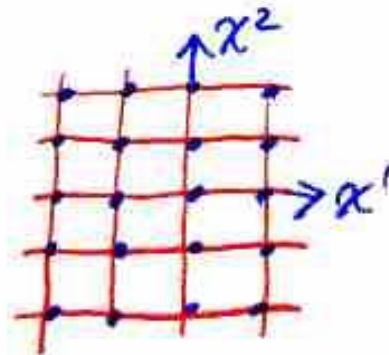
- 1930 Einstein attempted to include Torsion (Cartan 1922), but complained that he could not derive how particles moved in it.
- 1996 Kleinert finally solves it (*autoparallel* paths, not geodesics!)

- It explains Ehrenfest, Sagnac and Field Rotation Paradoxes
- Unlike curvature, hard to visualize.

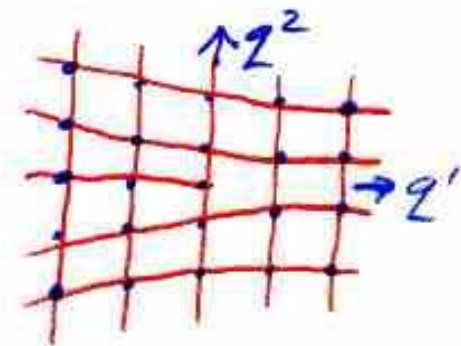
Kroner (1950s) showed that the Frank angle of crystal disclinations is equivalent to curvature, and Burges vector of dislocation is a measure of torsion.



Curved
(Disclination)



Flat



Torsion
(Dislocation)

Part III: Summary

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If rotational relativity is wrong:

What is absolute rotation measured against?

OR

If rotation IS relative (Mach is correct)

Paradoxes suggest must include torsion

Can torsion be used to explain away dark matter and dark energy?

IV. Is $E=mc^2$ wrong?

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A. Equivalence Principle Violations

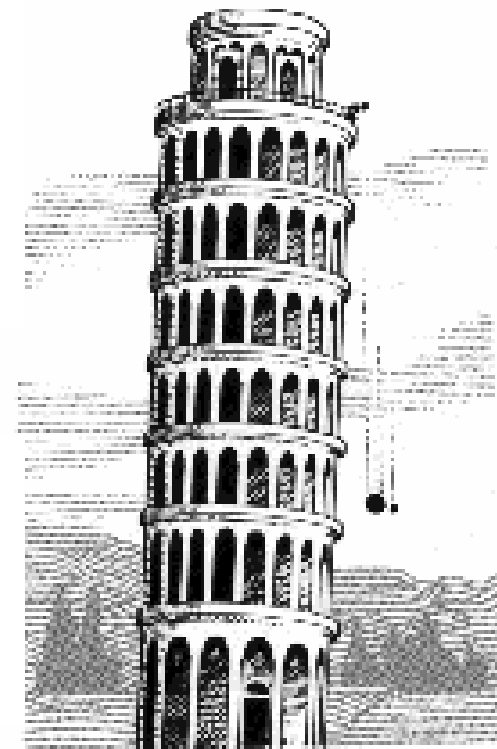
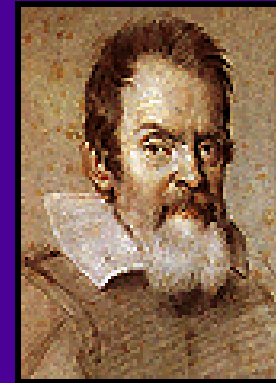
B. New Action Principle

C. Speculations

A1. Galileo's Experiment at Pisa

45

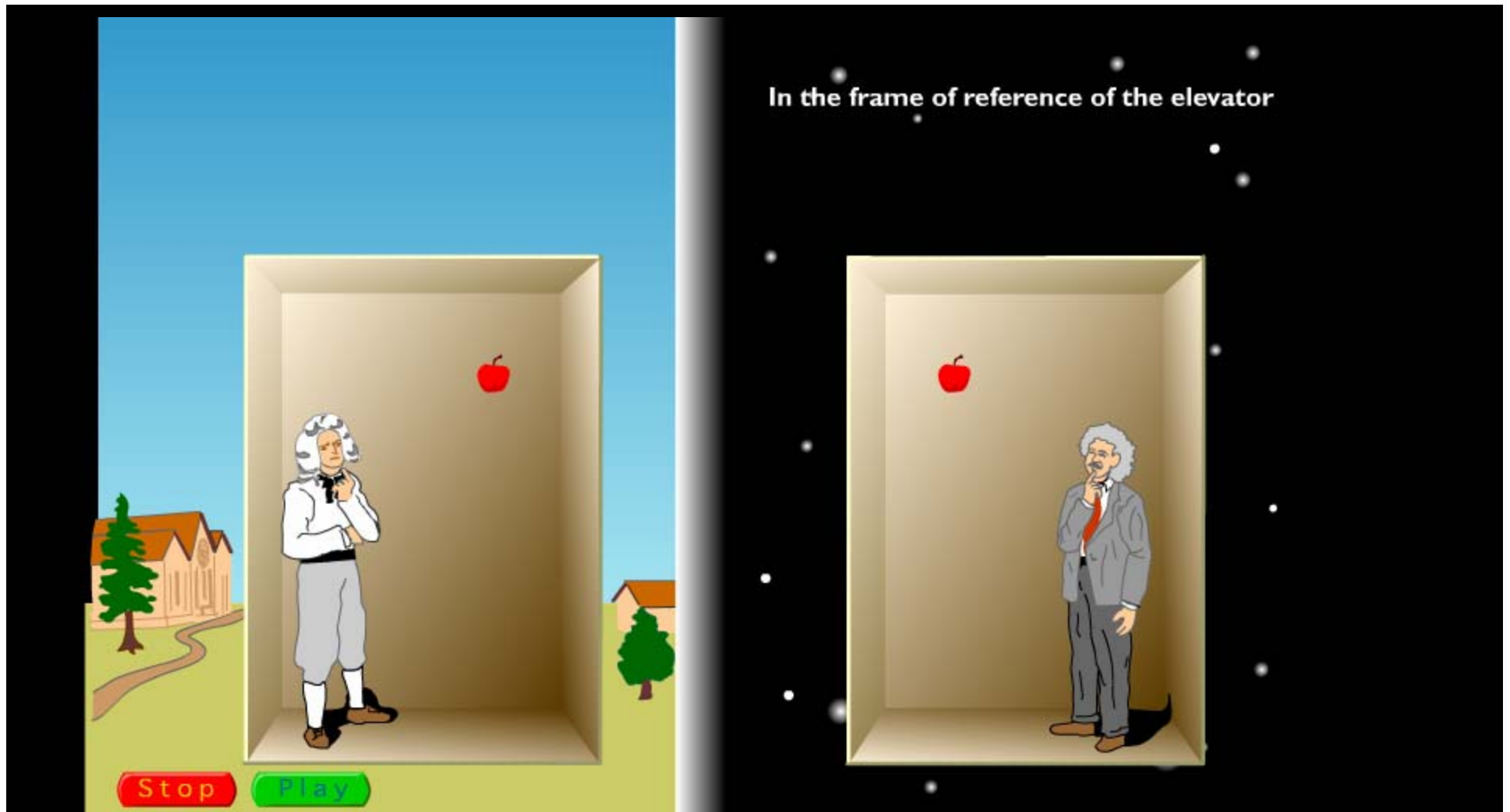
- **1590 Galileo's Principle:**
All bodies fall at the same rate, regardless of mass
- **1907 Weak EEP**
(Einstein Equivalence Principle)
All bodies will follow same path, independent of internal structure (e.g. mass or composition)
- **According to these, a spinning gyroscope should fall the same as a non-spinning one.**



A2. The Strong Equivalence Principle (1907)

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Reference Frame at rest with Gravity is indistinguishable to a reference frame which is accelerating upward in gravity free environment.



The apple accelerating downward due to gravity looks the same as an apple at rest in space, with the floor accelerating upward towards it.

A3. Spinning Particles Violate (Weak) EEP

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- (1951) Papapetrou Equations:

Shows spinning particles will deviate from “geodesics”

$$\dot{p}^\sigma + \dot{x}^\mu p^\nu \Gamma_{\mu\nu}^\sigma + \frac{1}{2} \dot{x}^\omega S^{\mu\nu} R_{\mu\nu\omega}{}^\sigma = 0$$

- Therefore, a spinning body will not fall the same as a non-spinning one.
- Violates Weak EEP.
(but not Strong EEP)



A4. Frenkel Equations (1926)

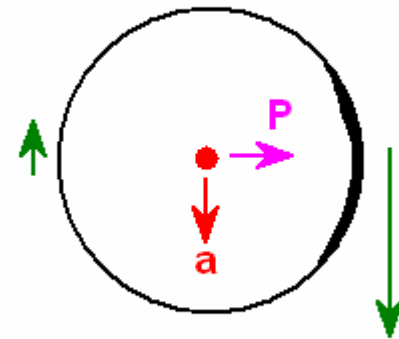
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Spinning Particles don't behave the way you expect

As it accelerates, the speed increases on the right, but decreases on the left

Higher speed increases the mass on the right side

Causes sideways contribution to momentum
(hence follows Strong EEP)



Newton's Momentum

Formula must be modified:

$$\vec{P} = m\vec{V} + \frac{\vec{a} \times \vec{S}}{c^2}$$

A5. Spin-Geometrodynamics

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- **Would the photon's spin cause it to take a different path? (Would the deviation change our estimates of sizes of things very distant?)**
- **Would there be a contribution to the redshift of a photon due to spin?**
- **Would a spinning spacecraft (e.g. Pioneer 10) have a little extra force on it?**

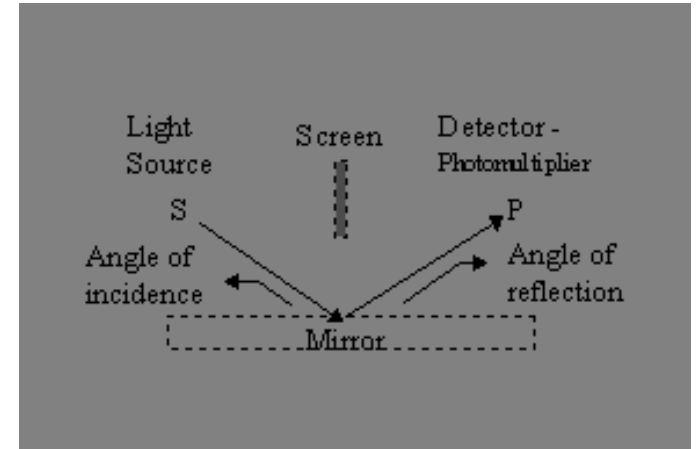
Yes to all, but the effect is too small?

B1. Least Distance

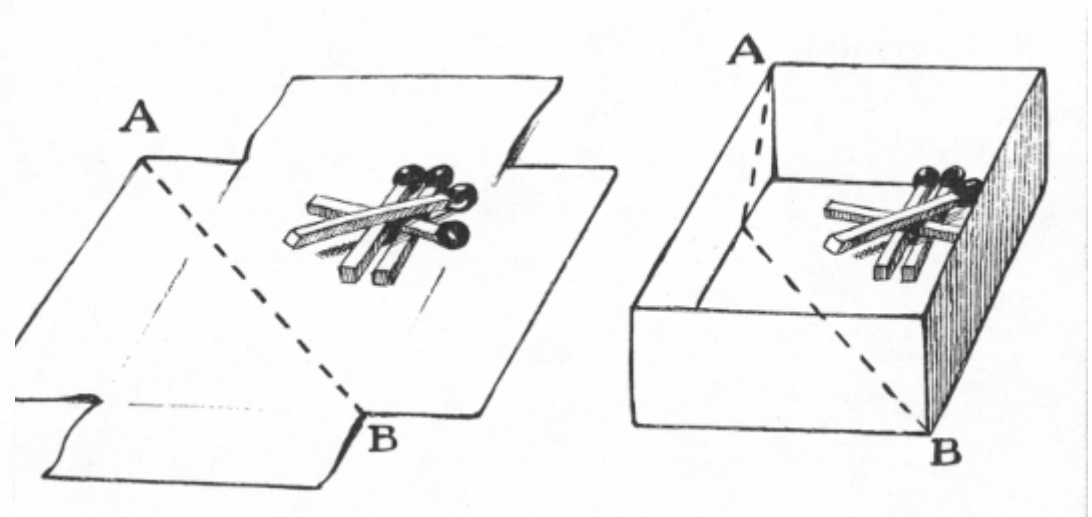
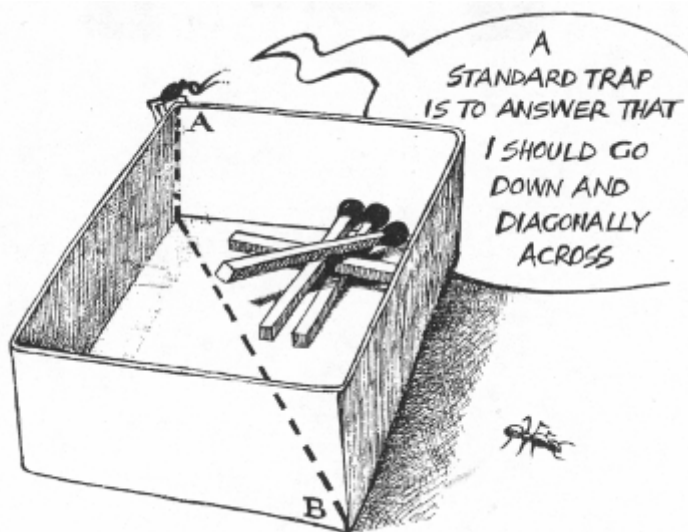
50

- Heron of Alexandria

Light follows path of least distance
(e.g. when reflecting off of water)



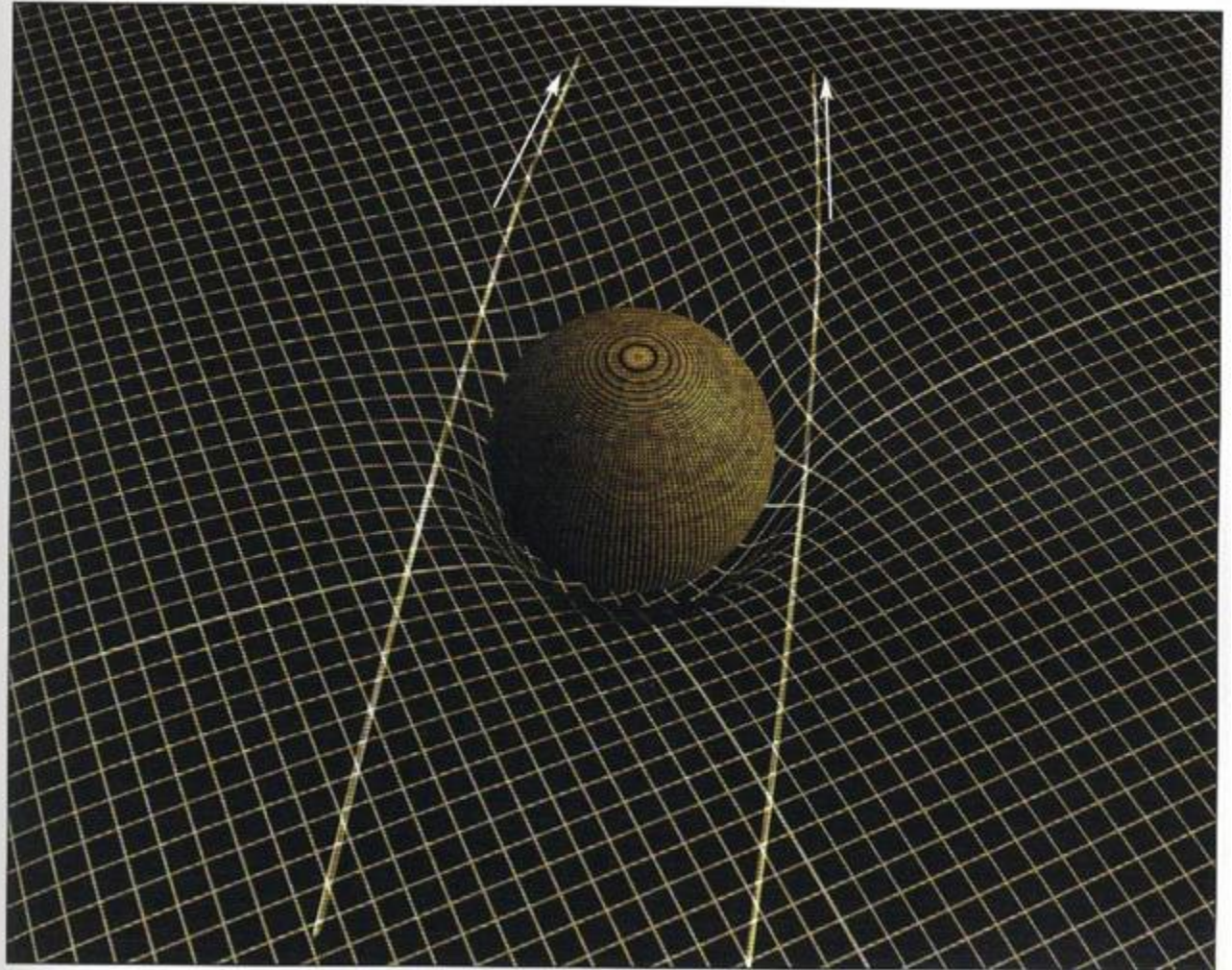
- Which path should an ant take to get to the opposite end of the box fastest?



B2. Geodesics

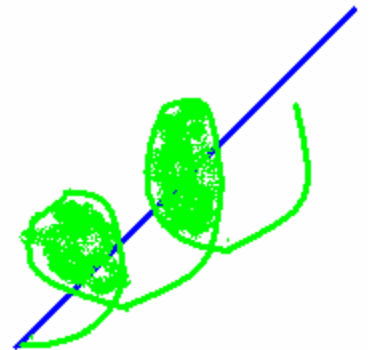
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In curved space, particles follow paths of least distance in 4D spacetime, called **geodesics**



But this doesn't give the right answer for spinning particles (or if there is torsion)

- (1998) I proposed that spinning particles follow paths which minimize the sum of:
 - Distance traced out by momentum
 - Area traced out by spin
- *I can derive the correct Papapetrou equations from this idea, even if the space is curved with torsion.*



B4. Finally: $E=mc^2$

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- Energy = mass x (speed of light)²
- *This is strictly true only if the particle is at rest.*
- *When it moves, it gains mass from kinetic energy*
(rest mass) = (total mass) - (Kinetic Energy)
- We usually write it this way:

$$(mc^2)^2 = E^2 - (Pc)^2$$

E=Energy

P=momentum (mass x velocity)

m=rest mass

B5. $E=mc^2$ is wrong?

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- 1970 Dixon showed that the formula must be modified for spinning objects.

$$(mc)^2 = p_\mu p^\mu - \frac{S_{\mu\nu} S^{\mu\nu}}{2\lambda^2}$$

- S is the spin angular momentum
 - λ is “radius of gyration”
 - m is the “non-spinning, non-moving” mass
 - *An increase in spin will increase the mass, which will in turn slow the particle’s velocity!*
-

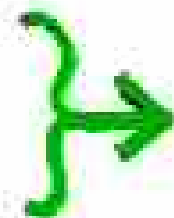
C1. Unify Phenomena with 4th Dimension

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- Combine Scalar law with Vector law with 4-vectors

- Scalar: $\dot{\mathcal{E}} = e \vec{E} \cdot \vec{v}$

- Vector: $\dot{\vec{p}} = e(\vec{E} + \vec{v} \times \vec{B})$

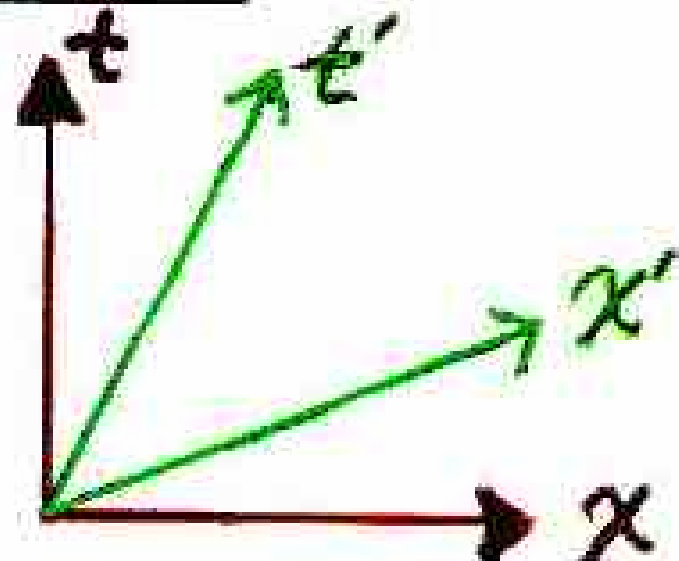


$$\dot{p}^\mu = \frac{e}{m} p_\nu F^{\mu\nu}$$

- Whether energy is a scalar or fourth component of a vector is relative to your point of view (3D or 4D reference frame).

Metaprinciple: Motion is Relative

- Laws same in every inertial frame.
Invariant under Lorentz Trans.
- What is (scalar) “time” to one observer is combination of space (vector) and time (scalar) to another.

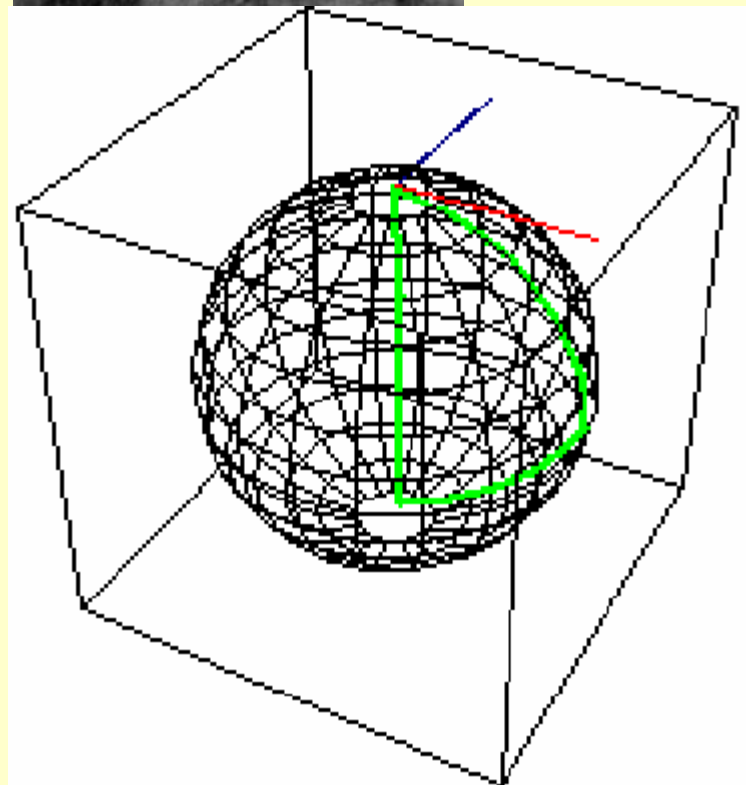


Levi-Civita (1917)

[two years after Einstein's General Relativity]

showed that a vector “**parallel transported**” around a closed loop will be rotated due to curvature.

We experience the rotation
(*holonomy angle*)
as **gravitational force**



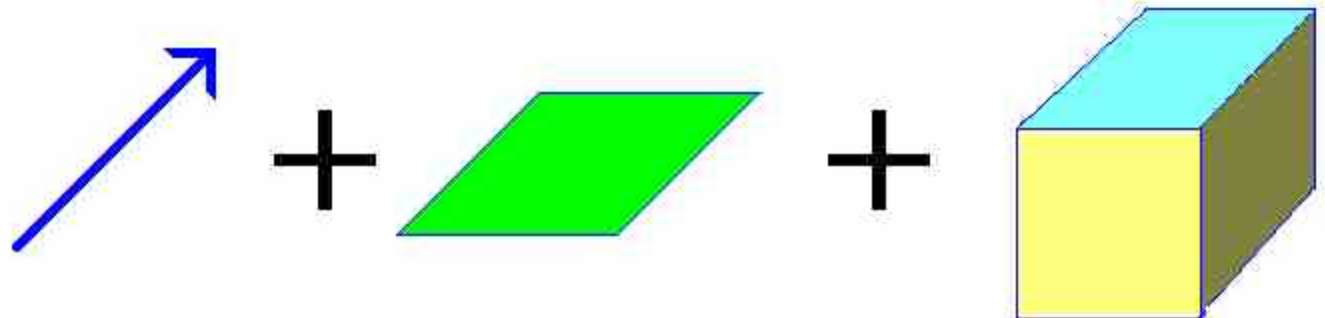
C3. William Kingdom Clifford (1876)

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1. *That small portions of space are in fact of a nature analogous to little hills on a surface which is on the average flat; namely, that the ordinary laws of geometry are not valid in them.*
2. *That this property of being curved or distorted is continually being passed on from one portion of space to another after the manner of a wave*
3. *That this variation of the curvature of space is what really happens in that phenomenon which we call the motion of matter, whether ponderable or ethereal*
4. *That in the physical world, nothing else takes place but this variation, subject (possibly) to the law of continuity.*

Clifford Algebra has “**Dimensional Democracy**”,
allowing you to add lines to planes






Using Clifford Algebra, get 2 equations in 1

$$\left. \begin{array}{l}
 \text{Line: } \dot{p}^\mu = \frac{e}{m} p_\nu F^\mu{}_\nu \\
 \text{Plane: } \dot{S}^{\mu\nu} = \frac{e}{m} (F^\mu{}_\sigma S^{\sigma\nu} - F^\nu{}_\sigma S^{\sigma\mu})
 \end{array} \right\} \dot{\mathcal{M}} = \frac{e}{2m} [\mathcal{M}, \mathbf{F}]$$

$$\text{Polymomenta: } \mathcal{M} \equiv \underbrace{p^\mu \hat{e}_\mu}_{\text{Line}} + \underbrace{\frac{1}{2} S^{\mu\nu} \hat{e}_\mu \wedge \hat{e}_\nu}_{\text{Plane}}$$

P is the momentum, **S** is the spin, **F** is the electromagnetic field

C5. Relative Dimensionalism

Scalar	+	1	1
Vector 	-	Y_0	Y_0
	+	Y_1	Y_1
	+	Y_2	Y_2
	+	Y_3	Y_3
Bivector 	+	Y_0Y_1	Y_0Y_1
	+	Y_0Y_2	Y_0Y_2
	+	Y_0Y_3	Y_0Y_3
	-	Y_1Y_2	Y_1Y_2
	-	Y_2Y_3	Y_2Y_3
	-	Y_3Y_1	Y_3Y_1
Trivector (Pseudovtr) 	+	$Y_0Y_1Y_3$	$Y_0Y_1Y_3$
	+	$Y_0Y_1Y_2$	$Y_0Y_1Y_2$
	+	$Y_0Y_2Y_3$	$Y_0Y_2Y_3$
	-	$Y_2Y_3Y_1$	$Y_2Y_3Y_1$
Pseudoscalar	-	$Y_0Y_1Y_2Y_3$	$Y_0Y_1Y_2Y_3$

- Transformations that reshuffle the geometry leave the polydimensional equation invariant.
- There is no absolute “direction” in the universe to which one can assign the geometry of “vector”
- What is vector to one observer is a bivector to another

C6. Transdimensional Curvature

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- The parallel transport of a 1D vector object (e.g. *momentum quantity*) around a closed loop might “rotate” the object into a 2D bivector (e.g. *spin quantity*)



- The total length of the polyvector is unchanged
Invariant: $(\text{length})^2 + (\text{area}/\lambda)^2$
- We experience this transdimensional curvature as **new forces** that couple to spin and momentum.

- Rank non-preserving Metamorphic Connection

$$\begin{aligned}
 d\mathbf{e}_\mu &= \left(dx^\alpha \Gamma_{\alpha\mu}^\nu + \frac{1}{2} da^{\alpha\beta} R_{\alpha\beta\mu}^\nu \right) \mathbf{e}_\nu \\
 &+ \frac{1}{2} \left(dx^\alpha \Omega_{\alpha\mu}^{\nu\sigma} + \frac{1}{2} da^{\alpha\beta} Q_{\alpha\beta\mu}^{\nu\sigma} \right) \mathbf{e}_\nu \wedge \mathbf{e}_\sigma
 \end{aligned}$$

- Induces new spin couplings

$$\begin{aligned}
 0 &= \dot{p}^\mu + p^\nu \dot{x}^\beta \Gamma_{\beta\nu}^\mu - \frac{1}{4m} S^{\omega\sigma} S^{\alpha\beta} Q_{\alpha\beta\omega\sigma}^\mu \\
 &+ \frac{1}{2} \dot{x}^\alpha \left(S^{\omega\nu} R_{\omega\beta\alpha}^\mu - S^{\omega\sigma} \Omega_{\alpha\omega\sigma}^\mu \right)
 \end{aligned}$$

- [OK, You'd be more impressed if I could write down some field equations and explain away dark matter]

V1. *Is it Almost Over?*

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The prevailing view about Einstein's theory is consistent with the following statement made by a Nobel prize winning physicist:

“The most important fundamental laws and facts of physical science have all been discovered, and these are now so firmly established that the possibility of their ever being supplemented in consequence of new discoveries is exceedingly remote.”

Albert Abraham Michelson

1903

(before relativity and quantum mechanics were invented)

V2. Epilog

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Talk will be posted at:

<http://www.clifford.org/wpezzag/talks.html>

Contact: wpezzag@clifford.org

2005Nov18, talk at CCSF

VI. References

Dark Matter

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