Galileo & Einstein were wrong?
SPINNING BALLS FALL SIDEWAYS

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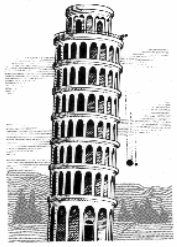


Outline

I. Introduction
II. Maxwellian Gravity
III. Spin & Inertial Mass
IV. Curvature
V. Summary
VI. References

Ia MetaThemes

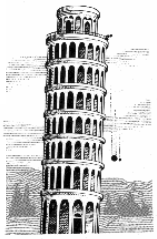
How do I talk about Einstein's theory without mathematics?

Fortunately, the **principles** of physics are not mathematical in origin.

Ib. Law of Falling Bodies

- **1590 Galileo's Principle:** All bodies fall at the same rate, regardless of mass

- **1907 Weak EEP** ***(Einstein Equivalence Principle)*** All bodies will follow same path, independent of internal structure (e.g. mass or composition)

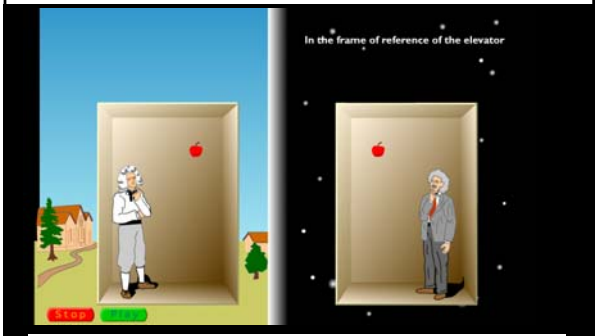


- **According to these, a spinning gyroscope should fall the same as a non-spinning one.**

WRONG

Ic. The Strong Equivalence Principle (1907)

Reference Frame at rest with Gravity is indistinguishable to a reference frame which is accelerating upward in gravity free environment.

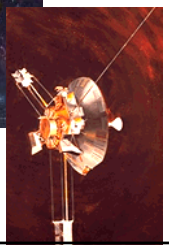


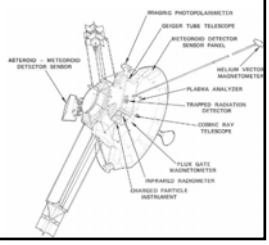
The apple accelerating downward due to gravity looks the same as an apple at rest in space, with the floor accelerating upward towards it.

Id. Pioneer 10/11 Anomaly

Launched: 1972/1973
Mission: Jupiter & Saturn

http://spaceprojects.arc.nasa.gov/Space_Projects/pioneer/PNhome.html





Ie. Pioneer 10/11

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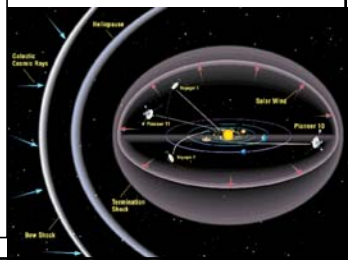
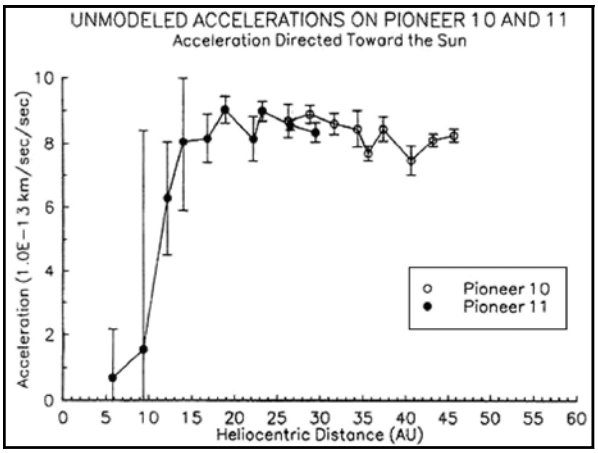
- **1983 Pioneer 10 at 30 AU, passes Neptune/Pluto, First spacecraft to leave solar system!**
- **1998 Pioneer 10 at 70 AU, traveling 2.5 AU/year**

1980 John Anderson, principle gravitation investigator, notes **anomalous acceleration toward sun**

1995 NASA funds grant to study

2002 Paper Published





If. Is Einstein's Theory Wrong?

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Consider Pioneer reached Neptune (30 AU) after 10 years

At 30 AU	Effect
Sun's Gravity	-1.5×10^{-5} m/s^2
Solar Wind	$< +\ 2\times10^{-10}$ m/s^2
Anomalous	-8.74×10^{-10} m/s^2
Change in range in 10 years due to effect	-0.003 AU

•Turyshev et al, AJP 73, 1033 (2005), "Study of the Pioneer anomaly: A problem set"

II. Maxwellian Gravity

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A. Gravity Analog of Electromagnetism

- 1864 Maxwell proposes gravitoelectric theory
- 1894 Heaviside (in 3 weeks) extends ideas to include gravitomagnetism

- 1915 Einstein's Theory

1. GravitoElectric Field

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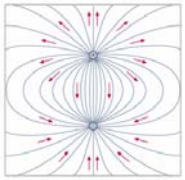
(a) Electric Field

- 1821 Faraday First proposes ideas of "Lines of Force"



- Force on Charge: $\vec{F} = q\vec{E}$
- Gauss's Law: $\nabla \bullet \vec{E} = \frac{\rho}{\varepsilon_0}$

(b) GravitoElectric Field is "g"

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Gravitational Analogy (to electric force):

- Force Law: $\vec{F} = m\vec{g}$
- "Field" is acceleration of gravity "g"
- "Charge" is (gravitational) mass "m"

(c) Gauss's Law for "g"

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- 1813 Gauss's Law $\nabla \bullet \vec{E} = \frac{\rho}{\varepsilon_0}$
- yields field around charge: $\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$

- Newton's Law implies "field" around mass: $\vec{g} = \frac{-GM}{r^2}\hat{r}$
- 1864 Maxwell writes: (1764 Lagrange) $\nabla \bullet \vec{g} = -4\pi G\rho$

2. GravitoMagnetic Field

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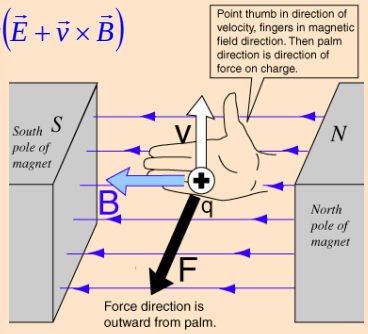
(a) Magnetic Force (1861 Maxwell)

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$



(b). Gravitational Analogy to Magnetic

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- 1889 Heaviside proposes "gravitomagnetic field" g_B

$$\vec{F} = m\left(\vec{g}_E + \vec{v} \times \vec{g}_B\right)$$

- g_B has not yet been directly verified! [more on *Gravity Probe B* later]

(c) Ampere's Law for "g_B"

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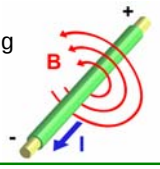
- 1856 Maxwell writes that moving charge creates magnetic field

$$\nabla \times \vec{B} = \mu_0 \rho \vec{v} + \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}$$



- 1894 Heaviside proposes that moving mass creates gravito-magnetic field (effect very small)

$$\nabla \times \vec{g}_B = -\frac{4\pi G}{c}\rho\vec{v} + \frac{1}{c^2}\frac{\partial \vec{g}_E}{\partial t}$$

B. GravitoMagnetic Moment

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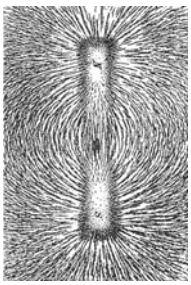
1. Magnetic Dipoles



(a) Magnets create fields

- Magnets have 2 poles
- No magnetic monopoles (why? Nobody knows!)

$$\nabla \bullet \vec{B} = 0$$

(b). Electromagnets

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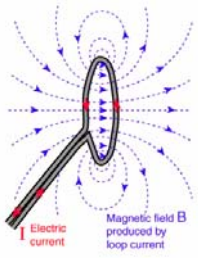
- 1823 William Sturgeon shows Current Loop is equivalent to a magnet
- Magnetic Moment (N turns, Current I, area A)

$$\mu = NIA$$



(c). Gyromagnetic Ratio

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- Rotating Ball of Charge (e.g. electron) should have magnetic moment proportional to spin:

$$\vec{\mu} = \gamma \vec{S}$$

- Gyromagnetic Ratio: $\gamma = \frac{ge}{2m}$
- Anomalous "g" factor
  - g=1 for classical objects
  - g=2 however for pointlike electron (why?)

2. Gravitomagnetic Moment is Spin

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- Rotating Ball of mass should have gravitomagnetic moment proportional to spin angular momentum $\vec{\mu} = g_G \vec{S}$
- Anomalous "g" factor value is disputed
  - g=1 for classical objects???
  - g=2 for pointlike electron????

3. Dipole Fields

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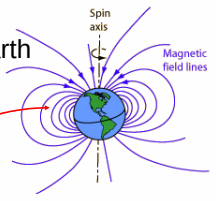
(a) Magnetic Field of Dipole

- At the equator, field is parallel to surface of earth with value

$$B(r) = \left(\frac{\mu_0}{4\pi}\right)\frac{\mu}{r^3}$$



(b). Gravitomagnetic Field of Earth

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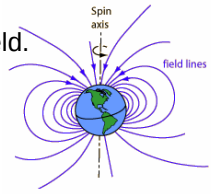
- In analogy, the spin angular momentum "L" of the earth creates gravitomagnetic field.
- At equator it is parallel to surface of earth with value

$$g_B = \left(\frac{G}{c^2}\right)\frac{L}{r^3}$$



(c). Balls fall sideways!

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- Acceleration on falling ball (equator of earth)

$$\vec{a} = \frac{\vec{F}}{m} = \vec{g} + \left(\frac{G}{c^2}\right)\frac{\vec{v}\times\vec{L}}{r^3}$$

- Einstein Equivalence Principle is valid! Big and small balls will travel same path.
- Homework #1: *After falling distance "d" how far sideways will a solid spinning ball go?*

C. Precession

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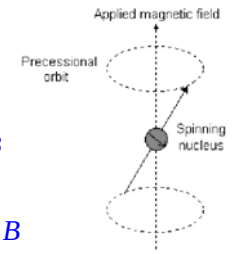
1. Larmor Precession

(a) Torque on Magnet $\vec{\tau} = \vec{\mu}\times\vec{B}$

(b) Precession $\frac{d\vec{S}}{dt} = \gamma\vec{S}\times\vec{B}$

(c) Larmor Frequency $\omega = \frac{\dot{S}}{S} = \gamma B$



2. Schiff Precession (1960)

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(a) Gravitomagnetic Torque on Spin $\vec{\tau} = \vec{S} \times \vec{g}_B$

(b) Precession equals gravitomagnetic field $\omega = \frac{\dot{S}}{S} = g_B$

(c) "Frame Dragging" precession due to angular momentum "L" of Earth $\omega = \frac{GL}{c^2 R^3}$

Homework #2: Calculate precession rate at equator of earth!

3. de Sitter Precession (1916)

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(a) Torque on Moving Magnetic Dipole $\vec{\tau} = \vec{\mu} \times \left(\vec{B} - \frac{1}{c^2} \vec{v} \times \vec{E} \right)$

(b) Gravitational Analogy $\vec{\tau} = \vec{S} \times \left(\vec{g}_B - \frac{1}{c^2} \vec{v} \times \vec{g}_E \right)$

(c) "Geodetic" precession due to gravitoelectric field $\frac{d\vec{S}}{dt} = -\frac{1}{c^2} \vec{S} \times \left(\vec{v} \times \vec{g} \right)$

Homework #3: Calculate precession rate for gyroscope in orbit around earth

4a. Gravity Probe B (Launched 2004)

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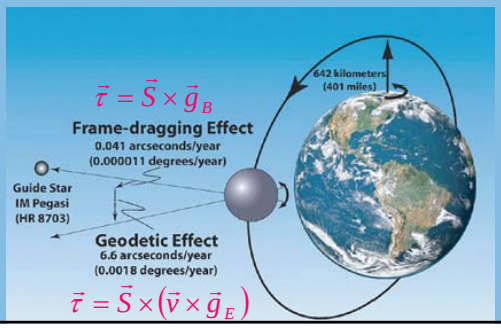
- Confirmed Geodetic to 1%
- still analyzing Frame-dragging (gravitomagnetic)



4b. The Kimball Experiment CSUEB

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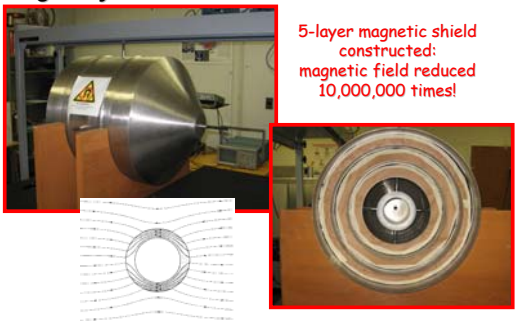
- **Attempt to measure nuclear spin precession due to gravity interaction**



III. Spin and Inertial Mass

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There are 3 ways to think about mass

1. Inertial Mass F=ma
2. Passive Gravitational Mass F=mg
3. Active Gravitational Mass $g = \frac{GM}{r^2}$

The "Weak Equivalence principle" says that inertial mass equals passive gravitational mass

Conventional view is that spin has no contribution to inertial mass (?)

A. Gradient Forces Violate EEP

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1. Gradient Forces

(a) Energy of Moving Magnetic Dipole $U = -\vec{\mu} \bullet \left(\vec{B} - \frac{1}{c^2} \vec{v} \times \vec{E} \right)$

(b) Gravity Analogy $U = -\vec{S} \bullet \left(\vec{g}_B - \frac{1}{c^2} \vec{v} \times \vec{g}_E \right)$

(c) Stern-Gerlach Force will depend upon spin! $\vec{F} = -\nabla U$

2. Gravitational Spin Orbit Coupling

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Moving Spin in gravitoelectric field violates EEP

- Gradient Force: $\vec{F} = -\frac{1}{c^2}\nabla\left(\vec{S}\bullet\vec{v}\times\vec{g}_E\right)$
- Will not modify falling motion $\vec{v}\parallel\vec{g}$
- But, Gyroscope with spin "S" in orbit with angular momentum "L" around static mass will experience spin dependent force

$$\vec{F} = \hat{r}\,g\left[-m\pm\frac{2\vec{L}\bullet\vec{S}}{mr^2c^2}\right] \qquad g=\frac{GM}{r^2}$$

3. Gravitational Spin-Spin Coupling

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- Gradient Force: $\vec{F} = -\nabla\left(\vec{S}\bullet\vec{g}_B\right)$
- Earth with angular momentum "L" creates gravitomagnetic field
- Force between spinning earth and Gyroscope of spin "S" is analogous to force between two magnets $\vec{F}=\frac{-3G\vec{L}\bullet\vec{S}}{c^2r^4}\hat{r}$

Spinning bodies will fall slower/faster !

Homework #4: How fast must a ball spin to be "weightless"?

B. Special Relativity

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1. **Spacetime Geometry**

(a) Unify Phenomena with 4th Dimension

Combine scalar law with vector law with 4 vectors

Scalar $\dot{E} = e\vec{E}\bullet\vec{v}$

Vector $\dot{\vec{P}} = e\left(\vec{E}+\vec{v}\times\vec{B}\right)$

$\Big\}\ p=\frac{e}{m}\,p\bullet F$

4 vector: $\mathbf{p} = p^1\hat{e}_1 + p^2\hat{e}_2 + p^2\hat{e}_3 + (E/c)\hat{e}_4$

Field Bivector $F = E^1\hat{e}_4\wedge\hat{e}_1 + E^2\hat{e}_4\wedge\hat{e}_2 + E^3\hat{e}_4\wedge\hat{e}_3 + B^1\hat{e}_2\wedge\hat{e}_3 + B^2\hat{e}_3\wedge\hat{e}_1 + B^3\hat{e}_1\wedge\hat{e}_2$

1(b). Grassmann Algebra

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Hermann Grassmann (1809-1877)

Inventor of "Linear Algebra"

1844 publishes massive work (which nobody understands) [1 year after quaternions!]

1c. Each Dimension is represented

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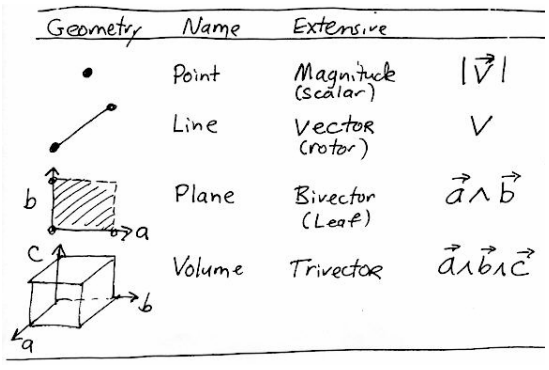

2. Rest Mass

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- Length of 4-momentum gives the rest mass of the particle

$$-(m_0c)^2 = p\bullet p = \vec{P}^2 - \left(\frac{E}{c}\right)^2$$

- Note "metric" of time part is negative
- Motion increases inertial mass: $E=mc^2$

$$m = m_0\sqrt{1+\left(\frac{\vec{P}}{m_0c}\right)^2} \approx \frac{1}{c^2}\left[m_0c^2 + \frac{\vec{P}^2}{2m_0} + \cdots\right]$$

Rest Energy — Kinetic Energy

3. 4D Rotational Invariance

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- There is no preferred direction in spacetime, hence laws of physics of physics must be form invariant under rotations
- Lorentz Transformations are rotations that reshuffle time and space (correspond to motion)
- Hence what is time (scalar) to one observer is combination of space (vector) and time (scalar) to another

- Lengths of 4-vectors invariant under rotations

C. Dynamic Mass

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1. **Relativistic Gradient Force equation** $\dot{p} = \frac{e}{m} p \bullet F - \Box(U \bullet F)$

- Is inconsistent with constant rest mass. Must include mass change: $\dot{p} = m\dot{v} + \dot{m}v$
- Contracting force equation with the four-velocity yields an equation for dynamic rest mass

$$\frac{dm_0'}{d\tau} = \frac{1}{c^2}\frac{d}{d\tau}\left[\vec{\mu} \bullet \left(\vec{B} - \tfrac{1}{c^2}\vec{v} \times \vec{E}\right)\right]$$

2. Mass changed by Dipole Interaction

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- Barut & others propose rest mass is hence:

$$m'_0 = m_0 + \tfrac{1}{c^2}\vec{\mu} \bullet \left(\vec{B} - \tfrac{1}{c^2}\vec{v} \times \vec{E}\right)$$

- Hence a dipole in a constant B field will have more inertia? (or less depending upon spin orientation). Has this ever been tested?
- In a big enough field, the mass would be ZERO?

Homework #5: How big of magnetic field would be needed for the rest mass of an electron to be zero?

3. Will mass be changed by Spin?

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Has anyone yet done the extension to Maxwellian Gravity? The analogous equation would be:

$$m'_0 = m_0 \pm \tfrac{1}{c^2}\vec{S} \bullet \left(\vec{g}_B - \tfrac{1}{c^2}\vec{v} \times \vec{g}_E\right)$$

- Hence spin-field interaction would change the **inertial** rest mass of particle
- Would it change the **gravitational mass** as well?
- Will it cause spinning balls to fall slower or faster?

IV. Curvature

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Einstein's General Theory of Relativity

Gravity is "curved space"

Big curvature makes a **Black Hole** that you can fall in and never get out

For example, when people throw things into my tuba, they are never seen again.

A1. The Holonomy of Curvature

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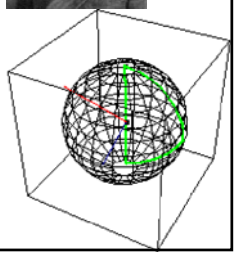
Levi-Civita (1917)
[two years after Einstein's General Relativity]

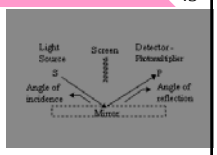
showed that a vector "parallel transported" around a closed loop will be rotated due to curvature.



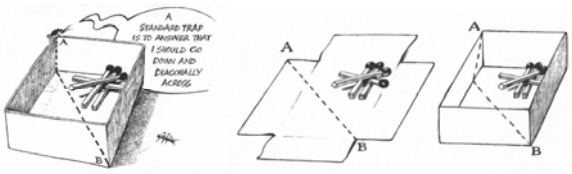
We experience the rotation *(holonomy angle)* **as gravitational force**

A2. Least Distance

- **Heron of Alexandria**
  Light follows path of least distance (e.g. when reflecting off of water)

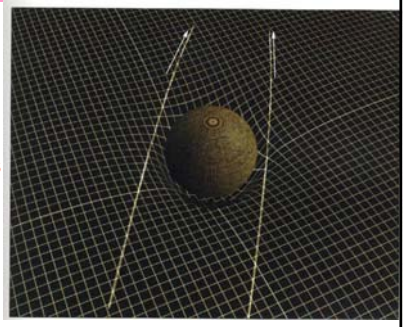


•Which path should an ant take to get to the opposite end of the box fastest?



A3. Geodesics

In curved space, particles follow paths of least distance in 4D spacetime, called geodesics



But this doesn't give the right answer for spinning particles (or if there is torsion)

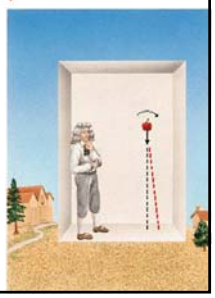
A4. Spinning Particles Violate (Weak) EEP

- **(1951) Papapetrou Equations:**
  Shows spinning particles will deviate from "geodesics"

$$\dot{p}^\sigma + \dot{x}^\mu p^\nu \Gamma^\sigma_{\mu\nu} + \tfrac{1}{2}\dot{x}^\omega S^{\mu\nu} R_{\mu\nu\omega}{}^\sigma = 0$$

Geodesic Part EEP violating

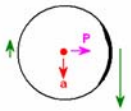
- **Therefore, a spinning body will not fall the same as a non-spinning one.**
- **Violates Weak EEP.**
  (but not Strong EEP?)



A5. Frenkel Equations (1926)

Spinning Particles don't behave the way you expect

As it accelerates, the speed increases on the right, but decreases on the left

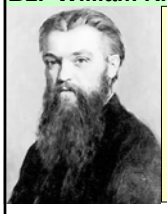


Higher speed increases the mass on the right side

Causes sideways contribution to momentum
(will this save the Strong EEP for falling gyroscope?)

Newton's Momentum Formula must be modified: $\vec{P} = m\vec{V} + \frac{\vec{a}\times\vec{S}}{c^2}$

B1. William Kingdom Clifford (1876)



1. *That small portions of space are in fact of a nature analogous to little hills on a surface which is on the average flat; namely, that the ordinary laws of geometry are not valid in them.*
2. *That this property of being curved or distorted is continually being passed on from one portion of space to another after the manner of a wave*
3. *That this variation of the curvature of space is what really happens in that phenomenon which we call the motion of matter, whether ponderable or ethereal*
4. *That in the physical world, nothing else takes place but this variation, subject (possibly) to the law of continuity.*

Clifford Algebra has "**Dimensional Democracy**", allowing you to add lines to planes

B2. Unify Phenomena Dimensionally

Using Clifford Algebra, get 2 equations in 1

Line: $\dot{p}^\mu = \frac{e}{m} p_\nu F^\mu{}_\nu$

Plane: $\dot{S}^{\mu\nu} = \frac{e}{m}\left(F^\mu{}_\sigma S^{\sigma\nu} - F^\nu{}_\sigma S^{\sigma\mu}\right)$

$\Big\}\ \dot{\mathcal{M}} = \frac{e}{2m}[\mathcal{M}, \mathrm{F}]$

Polymomenta: $\mathcal{M} \equiv \underbrace{p^\mu \hat{e}_\mu}_{Line} + \underbrace{\tfrac{1}{2} S^{\mu\nu} \hat{e}_\mu \wedge \hat{e}_\nu}_{Plane}$

P is the momentum, **S** is the spin, **F** is the electromagnetic field

B3. Spin Mass ($E=mc^2$ is wrong?)

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- 1970 Dixon shows formula must be modified

$$(m_0 c)^2 = p_\mu p^\mu - \frac{S_{\mu\nu} S^{\mu\nu}}{2\lambda^2}$$

- m is the "non-spinning, non-moving" mass, S is the spin angular momentum and λ is "radius of gyration"
- *An increase in spin will increase the mass, which will in turn slow the particle's velocity!*

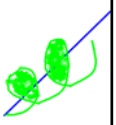
$$m = m_0 \sqrt{1 + \left(\frac{S}{m_0 c \lambda}\right)^2} \approx \frac{1}{c^2}\left[m_0 c^2 + \frac{\vec{S}^2}{2 m_0 \lambda^2} + \cdots \right]$$

Rest Energy — Spin Energy

B4. PolyGeodesics

50

- **(1998) I proposed that spinning particles follow paths which minimize the sum of:**
  - **Distance traced out by momentum**
  - **Area traced out by spin**



- *I can derive the correct Papapetrou equations from this idea, even if the space is curved with torsion.*
- *(2002) I get new force and torque terms when particle interacts with gravity+electromagnetism (unpublished)*

C1. Relative Dimensionalism

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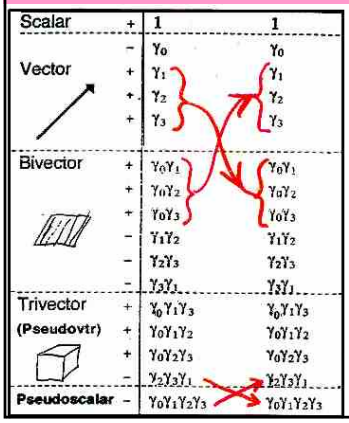
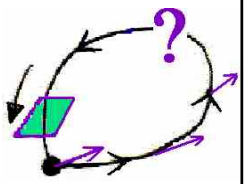


- **Transformations that reshuffle the geometry leave the polydimensional equation invariant.**
- **There is no absolute "direction" in the universe to which one can assign the geometry of "vector"**
- **What is vector to one observer is a bivector to another**

C2. Transdimensional Curvature

52

- The parallel transport of a 1D vector object *(e.g. momentum quantity)* around a closed loop might "rotate" the object into a 2D bivector *(e.g. spin quantity)*



- The total length of the polyvector is unchanged Invariant: **(length)² + (area/λ)²**
- We experience this transdimensional curvature as **new forces** that couple to spin and momentum.

C3. Details for 3 people in the audience

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- Rank non-preserving Metamorphic Connection

$$\mathbf{de}_\mu = \left(dx^\alpha \Gamma^\nu_{\alpha\mu} + \tfrac{1}{2} da^{\alpha\beta} R_{\alpha\beta\mu}{}^\nu \right) \mathbf{e}_\nu + \tfrac{1}{2}\left(dx^\alpha \Omega^{\nu\sigma}_{\alpha\mu} + \tfrac{1}{2} da^{\alpha\beta} Q_{\alpha\beta\mu}{}^{\nu\sigma} \right) \mathbf{e}_\nu \wedge \mathbf{e}_\sigma$$

- Induces new spin couplings

$$0 = \dot{p}^\mu + p^\nu \dot{x}^\beta \Gamma^\mu_{\beta\nu} - \tfrac{1}{4m} S^{\omega\sigma} S^{\alpha\beta} Q_{\alpha\beta\omega\sigma}{}^\mu + \tfrac{1}{2}\dot{x}^\alpha \left(S^{\omega\nu} R_{\omega\beta\alpha}{}^\mu - S^{\omega\sigma} \Omega_{\alpha\omega\sigma}{}^\mu \right)$$

- [OK, You'd be more impressed if I could write down some field equations and explain away dark matter]

V. Summary

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The prevailing view about Einstein's theory is consistent with the following statement made by a Nobel prize winning physicist:

> *"The most important fundamental laws and facts of physical science have all been discovered, and these are now so firmly established that the possibility of their ever being supplemented in consequence of new discoveries is exceedingly remote."*

Albert Abraham Michelson

1903
(before relativity and quantum mechanics were invented)

V. Summary

William Kingdon Clifford

(1845-1879)

… for geometry, you know, it the gate of science, and the gate is so low and small that one can only enter it as a little child.

VI. References

Slides of talk (along with all my other talks) available at:
`http://www.clifford.org/wpezzag/talks.html`

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